THE STUDENT'S EDITION

SIMILARITIES IN WAVE BEHAVIOR

BY DR. JOHN N. SHIVE
DIRECTOR OF EDUCATION & TRAINING
BELL TELEPHONE LABORATORIES

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PREPARED BY BELL TELEPHONE

WAVE BEHAVIOR

DR. JOHN N. SHIVE
DIRECTOR OF EDUCATION AND TRAINING
BELL TELEPHONE LABORATORIES, INC.

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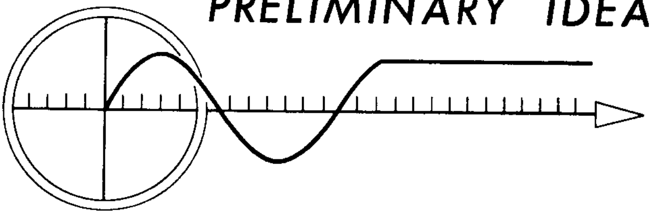
Contents

PRELIMINARY IDEAS



Everywhere we turn, we find waves of various kinds. Light and sound waves are probably the most familiar of these because their effects are so readily perceptible to our senses. Most of us are also familiar with water-surface waves. Other waves, having tremendous importance to our lives, are electrical waves on power lines and telephone wires, radio and TV waves.

The complete involvement of such waves in nature's vast scheme of things gives us ample cause to learn more about them. We soon see that they are fascinating in their own right, as well as highly instructive in the empirical sense because of their numerous applications in physics and engineering.

But, before we get any deeper into the subject, let's stop long enough to consider a definition of the beast. *A wave is a traveling disturbance of a medium from its normal condition.*

Now let's examine this definition piece by piece. The medium is the thing in which (or along which) the wave travels. Examples of such media are water surfaces, air, and wires, or just empty space.

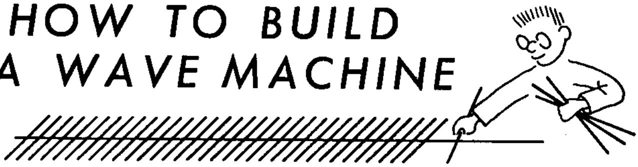
Let's begin with water surfaces. The normal condition of a water surface is to lie smooth and level.

However, we can alter this condition by tossing a stone into the water. This simple act creates a disturbance in the form of a series of concentric crests and troughs. The disturbance then travels outward from the center where the stone fell. It's important to note that the water doesn't travel with the waves. Only the condition of disturbance — that is, the wave itself, moves along. The water remains substantially where it was, with various portions of its surface merely bobbing up and down as the waves go by.

Now for another example. Take air in a room: Its normal condition is to be at a uniform pressure everywhere. However, if someone strikes a bell in the room, the bell's vibrations are communicated to the air layers adjacent to it. These layers, in turn, transmit the vibrations to the next farthest layers, and so on. The air vibrations are transmitted outward from the bell with the speed of sound, causing fluctuations of pressure and density in the successive air layers through which the sound passes. Such fluctuations comprise the disturbance of the medium (air) from its normal condition.

From the foregoing, it should be evident that waves in various media are caused by various disturbance-producing mechanisms.

HOW TO BUILD A WAVE MACHINE



The wave machine consists of a backbone-and-crosspiece structure supported in bearings which permit

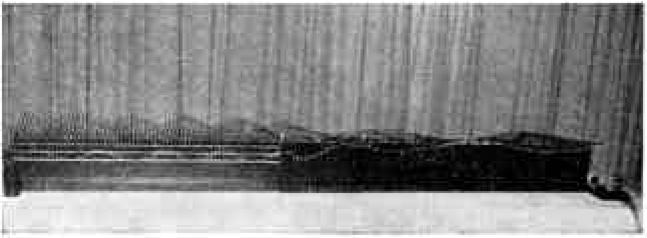


Figure 1. The wave machine completely assembled.

the structure to oscillate. Figure 1 shows a photograph of the machine completely assembled with a single wave traveling along it. Figure 2 is an exploded-view drawing of the machine showing the various parts separately and indicating how these parts are put together in the completed assembly.

In my machine the central backbone is a 3-foot long piece of 0.042-inch steel drill rod. The crosspieces are 18-inch lengths of 0.150-inch steel drill rod. These are soldered to the backbone exactly at their centers, parallel with each other and at right angles to the backbone. In the machine pictured in Figure 1, there are 70 crosspieces evenly spaced about ½ inch apart along the backbone.

There is nothing critical about the lengths or diameters of either the central backbone or the crosspieces. The larger the diameter of the central backbone, the faster the waves will travel along the machine. The larger the diameters and the longer the lengths of the crosspieces, the slower the wave will travel along the machine.

A neat job can be done of soldering the crossarms to the central backbone if a jig is used to insure even spacing. Such a jig (shown in Figure 3) is made by arranging small wire nails in the proper pattern on a

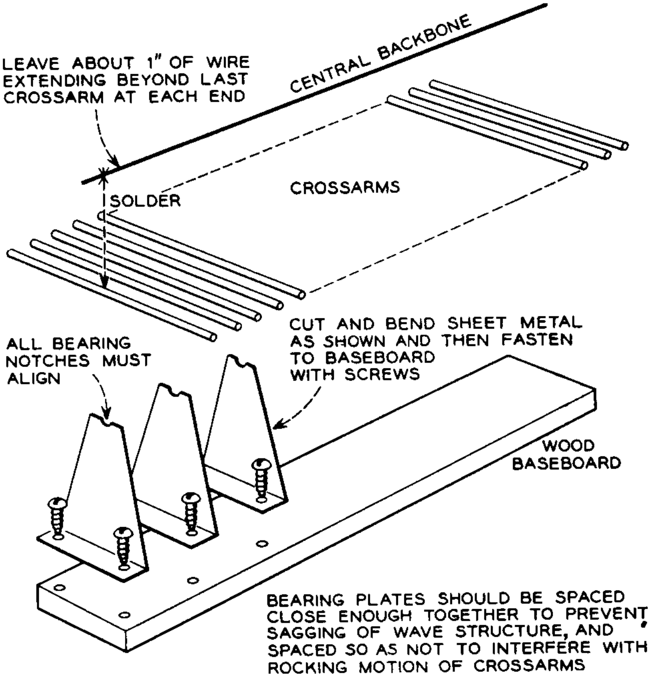


Figure 2. An exploded-view drawing of the wave machine.

plywood baseboard. One set of nails holds both the central backbone and the last soldered crossarm wire. Another set holds the next crossarm in proper relation to the rest of the structure while the solder connection is made. After each soldered connection is completed, the jig is moved one space farther along to set the stage for the next repetitive operation.

My friend, C. E. Briggs of the A.T. & T. Co., has devised what may turn out to be a simpler arrangement for holding the crossarms and backbone in proper relation to each other while solder connections are made. He took a 3-foot board, 6 or 8 inches wide, and cut grooves across the top surface, parallel to each other and perpendicular to the length of the board. The grooves were ½ inch apart and deep enough to hold the crossarms securely in place while the central wire was laid along the center line on top of the array and soldered to each crossarm. Mr. Briggs made quick work of slotting the board by using a circular saw. Figure 4 shows the jig board with a few of the crossarms placed for soldering.

Other Considerations: Mounting and Finishing

If you have any difficulty obtaining metal parts for the crosspieces, you might want to experiment with wooden dowel rods. These can be fastened to the central backbone with some of the iron glue cements available in hobby shops. Unfortunately, I'm afraid there's no escape from making the central backbone of metal.

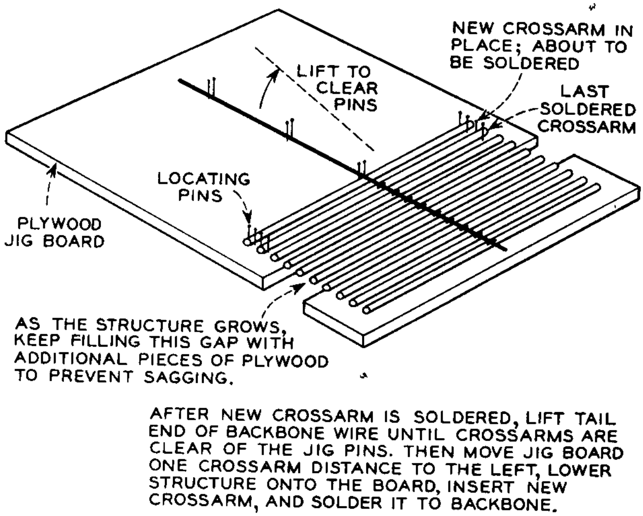


Figure 3. A simple jig which may be used in the construction of the backbone and crosspiece structure.

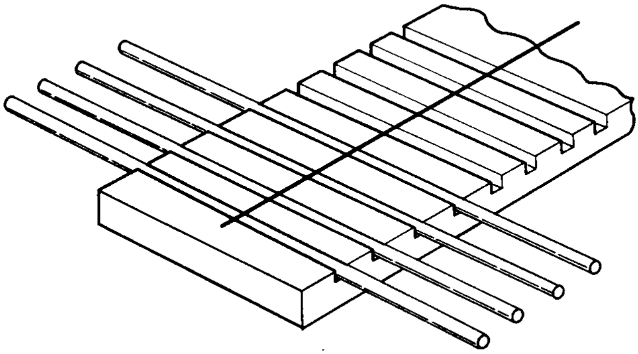


Figure 4. The jig devised by C. E. Briggs to aid in the assembly of the backbone and crosspiece structure.

Wood, plastic, stiff rubber tubing, and the like, may be springy enough, but they all exhibit internal hysteresis which would prevent the waves from traveling very far without being quickly attenuated because of energy absorption in the central backbone.

The wave structure itself rests in U-shaped notches in the top edges of the sheet metal bearing plates which are secured to the wooden baseboard as shown in Figure 2. The U-shaped slots should have a radius several times that of the central backbone. This allows the twisting central backbone to roll freely in the bearings when waves travel along the machine. The space between the bearing plates isn't critical; it should merely be small enough so the wave structure doesn't sag appreciably between bearings. The bearing plates should support the central backbone without coming into contact with any of the crossarms, or in any way interfering with their rocking motion.

After you have completed the fabrication of the wave structure in the jig, you will have to transfer it to the bearings of the base and mount. To do this with a minimum of damage, grasp the two ends of the machine where the central backbone wire is soldered to the first and last crosspieces. Then, without twisting or bending the central wire, lift the structure — simultaneously pulling outward on both ends to keep the medium from sagging beyond the elastic limit of the central wire. Lower the structure into the U-shaped notches in the bearing plates making sure the ends come out where they should. If you have properly located the bearing plates, they shouldn't interfere with the motion of the crossarms. Now you are ready to start a wave and watch it go.

In this manual, I will cover many features of wave behavior which can easily be demonstrated with your machine. Please keep in mind that, while the waves you produce and watch are mechanical, the principles discussed and demonstrated are equally valid for waves of all kinds. Nature exhibits a simplicity and universality which are beautifully borne out in the aspects of wave behavior we shall study.

Experiment 1

GETTING ACQUAINTED WITH WAVES

Start a single wave traveling along your machine. To do this gently pump the end of the first crossarm up and down once. The resulting wave will detach itself from your hand, run down to the other end of the machine, reflect and return, repeating the cycle several times before dying out. When you pump the end of the first crossarm up and down, what you are really doing is twisting and untwisting the end of the central wire to which the crossarm is attached. The wave is thus actually a wave of twisting and untwisting of the central wire, made perceptible to you by the action of the ends of the crossarms. The ends of the crossarms exhibit a transverse type of displacement, moving up and down for short distances while the wave itself travels horizontally along the structure.

Start another wave and ask yourself if it fulfills the wave definition which we considered earlier. Note that the portions of the machine, both ahead of the wave and behind it, are essentially at rest. The wave itself is thus a purely local affair which travels along the medium at its own good speed, this speed being determined by the physical properties of the medium.

In discussing wave behavior, we shall often make the distinction between single waves, or *pulses*, and continuous waves, which are long trains of single waves frequently repeated at equal intervals.

You can start a train of continuous waves on the wave machine by pumping the end of the first crossarm up and down repeatedly. If you do not already have a prepared wave-machine kit with a wave-generator motor, you may wish to make such a generator on your own. You can do this by taking an erector-set or electric-clock motor, gearing it down to a speed of one or two revolutions per second, attaching an eccentric crank to the shaft of the speed reducer, and then driving the first crossarm of the wave machine by means of the

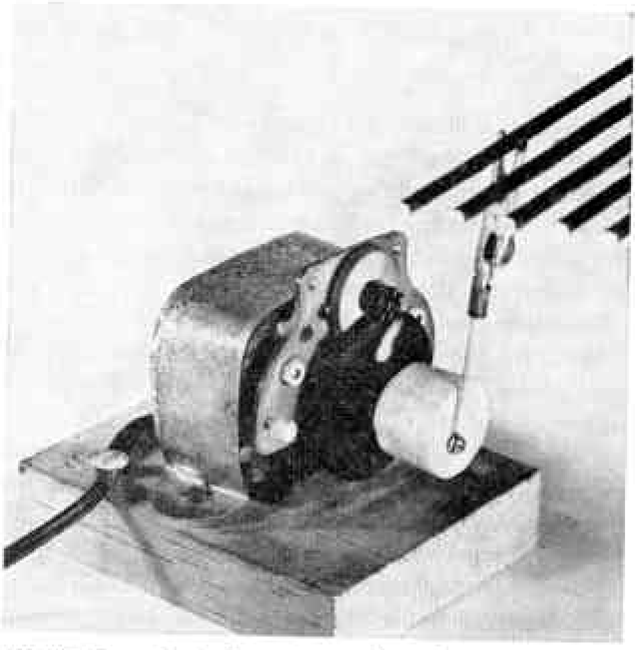


Figure 5. A simple motor and crank assembly used to generate continuous waves on the machine.

crank rod. The amplitude of the waves can be increased by moving the crank-rod clip farther along the crossarm toward the central wire. Figure 5 is a photograph of a synchronous midget motor with a speed-reducing gear train generating continuous waves.

Experiment 2

WAVE DAMPING

Perhaps one of the first things you'll notice about the waves on your machine is that they don't keep going forever. Generate some more waves on the machine and see how their amplitude decreases steadily until, after a few reflections between the ends of the machine, they disappear altogether.

This decrease in amplitude, or *damping*, is due to friction. In the mechanical wave machine there are three sources of friction each of which independently produces its share of damping. These are: (1) air friction operating on the moving crossarms, (2) rolling friction of the central wire twisting back and forth in its bearings, and (3) internal hysteresis friction in the central wire itself.

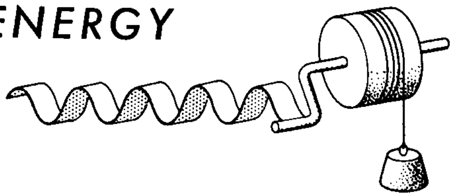
The damping of waves of all kinds is a fairly general phenomenon of nature.

Light waves are absorbed in a greater or lesser degree by almost everything they pass through, including objects which we ordinarily regard as transparent, such as air, water, and glass.

Electric waves on telephone wires and transmission lines are similarly damped because of the electrical resistance of the wires. As the waves travel along, the electrons in the wires surge back and forth. In this movement, they encounter frictional resistance which dissipates the energy of the wave in the form of heat. Without help, the electric waves which carry a telephone conversation along telephone lines die within 15 or 20 miles. Long-distance lines must be specially engineered to get around this difficulty. On such lines, the damping is compensated for along the way by amplifiers. These are inserted in the circuit every few miles in order to restore the original intensity of the waves.

Experiment 3

WAVES AS CARRIERS OF ENERGY



In the last section, we talked about the frictional processes that cause wave damping by abstracting energy from waves of various kinds. That waves transport energy can be demonstrated with your wave machine if you first make an auxiliary device like the one shown in Figure 6. This is simply a device for lifting weight by means of a ratchet arangement which is operated by the up-and-down motion of the last crossarm on the wave machine as continuous waves arrive from the generator. Figure 7 is a drawing of this apparatus. Enough detail is shown to enable you to make one like it.

The weight is raised a short distance on each stroke of the last crossarm. The altitude thus gained is held

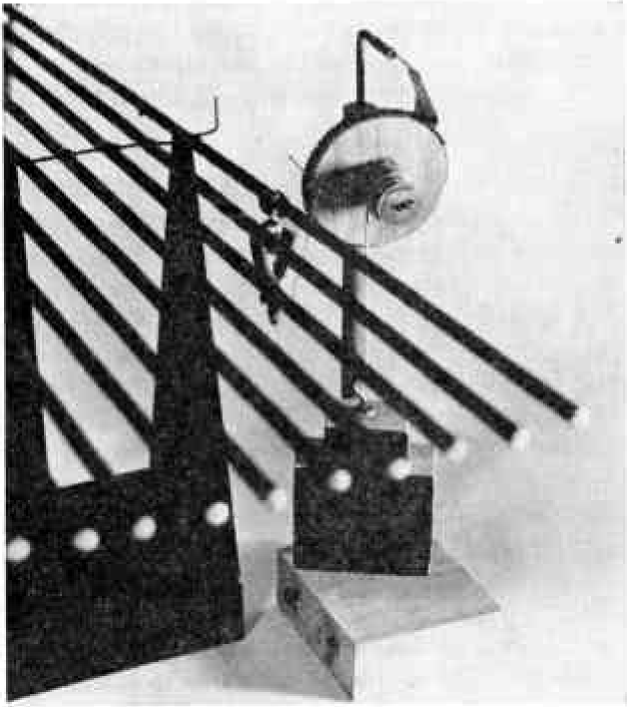


Figure 6. The ratchet lifting apparatus used to demonstrate that waves are carriers of energy.

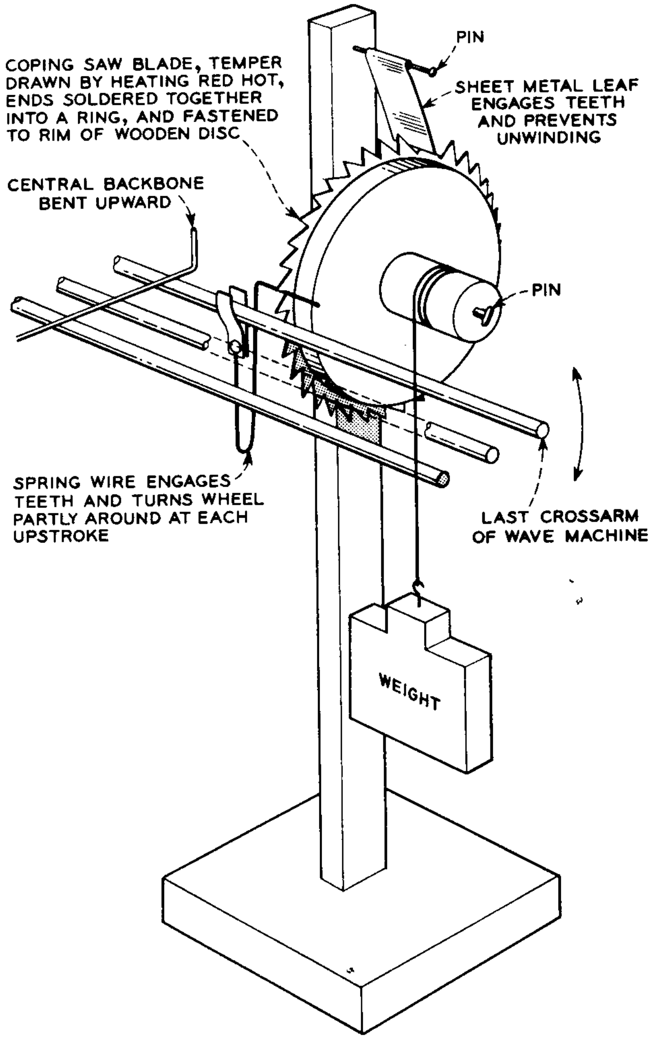


Figure 7. A detailed drawing of the ratchet lifting apparatus.

by the upper ratchet while the crossarm, with its ratchet spring wire, goes down for another lifting stroke. A little experimentation will show you how far out on the crossarm the weight-lifting attachment should be engaged for best results. Try the machine now and see how much work you can make a wave do.

While thinking of mechanical waves traveling along wave machines, you should also be thinking of sound waves traveling through air, electrical waves traveling along wires, and electromagnetic waves launched into space from radio and TV broadcast antennas. All these waves require energy to produce them, and they carry this energy with them as they travel along.

Experiment 4

WAVE SPEED

The speed with which a wave should travel on your wave machine can be calculated from a theoretically derived formula. It can also be determined by direct measurement with a meter stick and a watch. Shall we see how closely these two methods of speed determination agree?

To measure wave speed with a meter stick and a watch it is only necessary to record how much time it takes for a wave to traverse a certain measured distance along the machine. The quotient of distance and time will be the actual wave speed. That is:

$$s = d/t, \qquad (4\text{-}1)$$

where

s = the wave speed,
d = the distance traveled by the wave, and
t = the time required by the wave to travel distance d.

Measure the speed of a wave on your machine by the direct method described above. Record the results so that we may compare them with the results from theoretical computation of the wave speed.

By formula, the speed of a wave on the machine, s, should be:

$$s = \sqrt{\tau/I}, \qquad (4\text{-}2)$$

where

τ = torsion constant of the central wire, and
I = the rotational moment of inertia of the crossarm system per unit length along the machine.

τ may be found as follows. Clamp the end of the tenth crossarm of the wave machine so that it can't move. Then hang a small mass (m grams) from the end of the first crossarm so as to produce a 10 or 15 degree twist between the weighted crossarm and the clamped crossarm. Measure the angle of twist, θ, and convert it to radians. Now, measure the distance l in centimeters between the centers of the weighted and clamped crossarms, and measure the distance d from the central wire out to where the mass is hung on the first crossarm. Calculate τ from the expression:

$$\tau = 980 \frac{mdl}{\theta} \cos \theta. \qquad (4\text{-}3)$$

I may be found as follows. Determine the mass M in grams and length L in centimeters of one of the crossarms. Also measure the distance D between adjacent crossarms. Then calculate I from the expression:

$$I = \frac{1}{12} \frac{ML^2}{D}. \qquad (4\text{-}4)$$

Now, put these measured and calculated values of τ and I into equation (4-2), and find out how fast a wave should travel on your machine.

NOTE: For those who encounter difficulty in carrying out the foregoing computations because of the measurement units used, the following conversion table may be helpful.

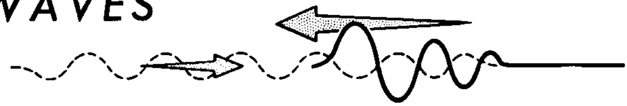
1 cm	2.54 inch
1 oz	28.35 gram
1 rad	57.3°

Now compare your results from the actual measurement of wave speed and the calculated determination of wave speed. Do the two speeds agree within your ability to measure the various quantities involved?

There is one thing implicit in the speed expression (4-2) which is worth pointing out. That is that the equation says nothing about the amplitude or wave length of the wave. The implication here is that waves of all sizes and shapes travel along the machine at identical speeds. Can this really be true? Try it for yourself and see. With your watch, find out how long it takes little waves, big waves, long waves, and short waves to travel the length of your machine. You'll find that these times are, in fact, all the same.

Experiment 5

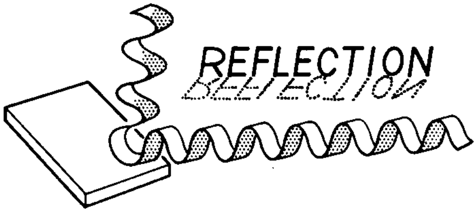
CRISSCROSSING OF WAVES



Waves going in different directions on the same medium pass right through each other. To demonstrate this generality, start two single waves simultaneously, traveling in opposite directions from each end of your wave machine. Now, watch what happens when they meet in the middle of the machine. They pass right through each other, don't they? Or . . . wait a minute, now. Do they pass through each other, or merely collide and rebound? Which wave is which?

You can resolve this uncertainty by starting two more waves toward each other from opposite ends of the machine. But, this time, make them of different amplitudes. In this way, you can keep track of each wave separately. Now, there's no guesswork about what happens, is there? The waves do *not* collide and rebound; they pass through each other (or climb over each other) and continue on their merry ways as if they had never met.

Experiment 6



Reflection is one of the most generally familiar aspects of wave behavior. Optical reflection in mirrors, and sound reflection from buildings or mountains (echoes), are all part of common experience. Less familiar, perhaps, but equally common, are reflections of electric waves on lines and reflections of certain radio waves from the ionosphere and even from artificial satellites such as the high-flying *Echo.*

Now, like Socrates, let's define our terms — or, at least, one of them. *Reflection is the turning back of a wave upon itself when it encounters an abrupt change in the nature of the medium in which it is traveling.*

Reflection may be partial or total, depending on the severity of the change. And it may or may not be accompanied by some absorption as well.

Start a wave on the machine and notice that the wave, having traveled the length of the machine, reverses its direction when it arrives at the end and then travels back to its starting point. At each reflection, the wave turns around bodily, preserving both its amplitude and shape. Since the discontinuity of the machine is extreme at the dead end, the reflection is total. There is no mechanism for extracting energy from the wave in this particular act of reflection. All the energy incident with the wave is, thus, reflected back with it.

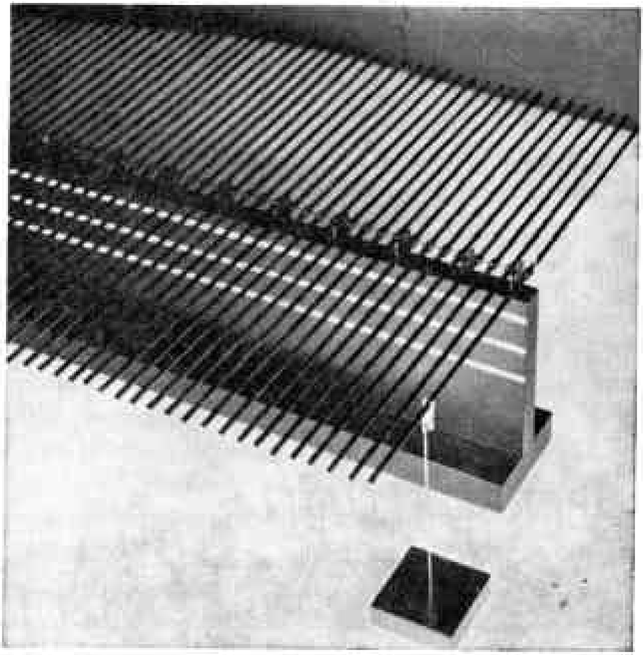


Figure 8. The last crossarm on the machine is clamped so that it cannot be displaced when a wave comes along.

Now, instead of leaving the reflecting end of the machine free as before, clamp the last crossarm so that it can't be displaced when a wave comes along (see Figure 8). Launch a wave and watch what happens when it encounters the clamped-end crossarm. The wave is totally reflected as before, but it is turned upside down in the process! Free end, right side up reflection; clamped end, upside-down reflection.

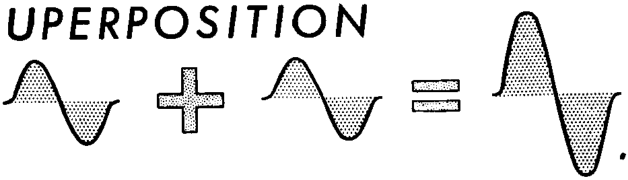
The same two possibilities for total reflection occur on other wave systems as well. In every case, the kind of reflection you get depends on whether the reflecting end of the medium is free or clamped.

For example, an open-end acoustic tube will act to reflect sound waves without inversion and a closed-end acoustic tube will cause the sound waves to be inverted when they are reflected.

Analogous behavior is found for total reflection at the end of an electric transmission line. If such a line is short-circuited at the far end, the reflected wave comes back without inversion. If the other end is left open-circuited, the returning reflected wave is inverted.

Experiment 7

SUPERPOSITION



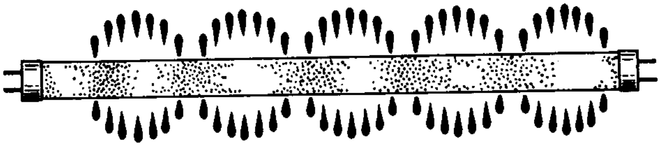
One of the underlying principles concerned in wave behavior which relates a number of different wave phenomena to a single unifying idea, is the principle of superposition. This principle states that when two waves meet each other on the same medium, the instantaneous displacement of the medium is given by the algebraic sum of the instantaneous displacements of the individual waves. Upon this principle depend a number of phenomena which we shall discuss. These phenomena include interference, standing waves, and resonance.

The validity of the principle can easily be demonstrated with your wave machine. Start two identical symetrically-shaped single waves traveling toward each other from the opposite ends of the machine. Notice that, when they meet in the middle, the amplitude of the single crest resulting at the instant of exact superposition is clearly larger than the individual amplitude of either wave. How can you prove this? One way is simply to observe the progression of the waves against a scale consisting of a number of parallel equally-spaced horizontal lines. Make a quick visible estimate of the amplitudes of the individual waves before they meet and a similar estimate of the resulting amplitude at the instant of exact coincidence. Is the latter the algebraic sum of the former?

The principle of superposition is equally valid for the superposition of waves having opposite directions of displacement. Again, start single waves simultaneously from opposite ends of the machine, making them as symmetrical in shape and equal in amplitude as you can, but, this time, make one with its displacement upward and the other with its displacement downward. When the waves meet in the middle of the machine, there will be an instant — the instant of exact coincidence — when the displacement of the machine is zero. This condition is fleeting to be sure. However, after the waves pass through each other, they resume their original shapes and continue their original independent ways unchanged by the experience.

Experiment 8

INTERFERENCE AND STANDING WAVES



Now that we've considered the superposition behavior of single waves or pulses, we are ready to talk about the interference of trains of continuous waves moving in opposite directions on the same medium. Let's see what happens if we start two trains of waves going in opposite directions on the machine. Interference should occur and a specific wave pattern will become apparent.

Two identical trains of waves going in opposite directions on the machine can be produced by simply generating a train of continuous waves at one end with the motor and crank attachment, and totally reflecting these waves back from the other end so that the going and returning wave trains pass through each other on the machine.

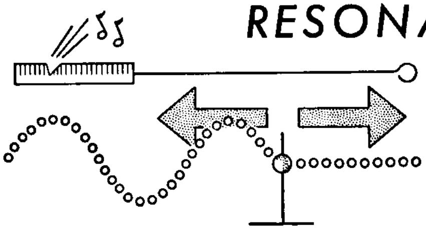
Soon a steady-state wave pattern will emerge which exhibits certain features. Note the peculiar appearance of this pattern. It consists of a number of segments separated by the nodes, which simply bob up and down in place without going anywhere along the machine.

These are standing waves — in contrast with running waves, which travel along the machine. The standing waves are formed by the superposition of running waves going in opposite directions. Just as the nodes are produced by the destructive interference of the waves in the two trains, the maximum amplitude of the loops midway between each pair of nodes is produced by the constructive interference of the waves in the two wave trains. At these midway points, the amplitude of the loops is twice the amplitude of the running waves in the individual wave trains. While the nodes appear at regularly-spaced intervals half a wave length apart, the position of the nodal pattern with respect to the machines ends depends on how you chose to produce the reflection at the terminating end. If you employed a free end as the reflector, the node nearest this end appeared a quarter of a wave length away toward the generator. But, if you used a clamp for the reflector, the clamped end became itself a node since the clamp enforced a condition of zero displacement. The next nearest node should, therefore, have appeared half a wave length away from the end.

Note that by using the clamped crossarm as a reflector, you can chase the entire pattern of nodes up the machine toward the generator by moving the position of the clamp in that direction. The location of the nodes depends on the nature and positioning of the reflector. The distance between the reflector and the generator merely determines how many nodes there will be.

Experiment 9

RESONANCE



While you were experimenting with different placements of the generator and reflector, you must have noticed that the amplitude of oscillation of the loops of the standing waves varied from case to case. Let us examine this variation systematically.

Attach the generator to the first crossarm throughout the following procedure. Now attach the clamp to the last crossarm, and note the standing wave amplitude which persists after the *transient* has died out. Then move the clamp just far enough up the machine to drive the node nearest the generator to within three or four crossarm spacings' distance of the generator crossarm. Again, note the standing-wave amplitude. Now move the clamp, a single crossarm at a time, closer to the generator, noting the steady-state standing-wave amplitude after each change (see Figure 9).

As the node is driven closer to the generator crossarm, the standing-wave amplitude increases. In fact, you'll have to be careful not to allow the amplitude to become so large the machine comes in danger of jumping out of its bearings, or of twisting the central wire beyond its elastic limit.

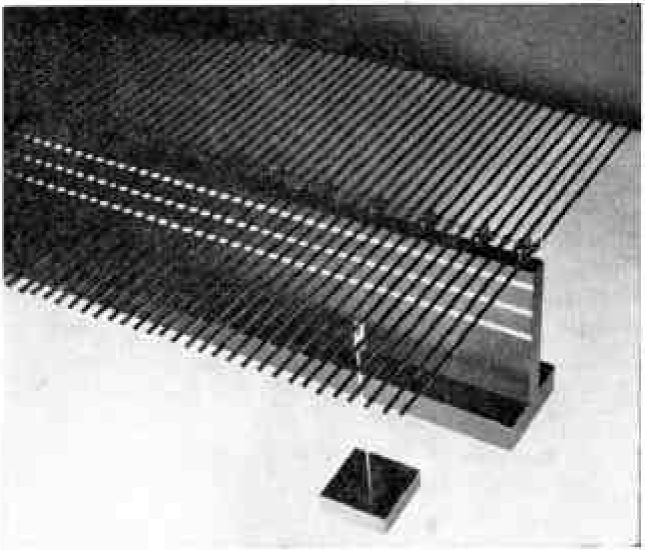


Figure 9. The clamp is moved, a single crossarm at a time, closer to the generator as we seek to find the point where the system is in a condition of resonance.

At this point, the system is in a condition of *resonance*. The abnormally large amplitude is an extreme example of superposition. The length of the machine is now equal to (or very nearly equal to) a whole number of half wave lengths. This correspondence means that the travel time of a wave from the generator down to the reflector and back again is a whole number of wave periods. Thus, a wave, starting at the generator and traveling back and forth alogg the machine between generator and reflector, always arrives back at the generator in just the right phase to superpose itself upon a new wave being sent out by the generator at that instant. The new waves and the previously emitted, multiple reflected waves thus superpose constructively, crest on crest, as they travel back and forth on the machine. Because of this constructive superposition, the amplitude builds up until the rate of frictional energy loss, which becomes larger as the amplitude increases, is equal to the rate of new energy input from the generator.

The process of adjusting the length of the wave machine to a whole number of standing-wave loops in order to produce resonance is called *tuning*. Tuning the wave machine to resonance could have been accomplished just as well by leaving the machine at its original full length and varying the frequency of the generator.

Experiment 10

THE Z IMPEDANCE CONCEPT

The process of wave propagation can be described in terms of two quantities — *cause* and *effect*. Here's an example: to launch a train of waves on your wave machine, you apply an oscillating torque which is the cause of the waves. Successive portions of the wave machine respond by executing an oscillating angular displacement and a corresponding oscillating angular velocity. This is the effect. In wave theory the ratio of cause to effect is termed "impedance". That is:

$$\text{Impedance} = \frac{\text{Cause}}{\text{Effect}}.$$

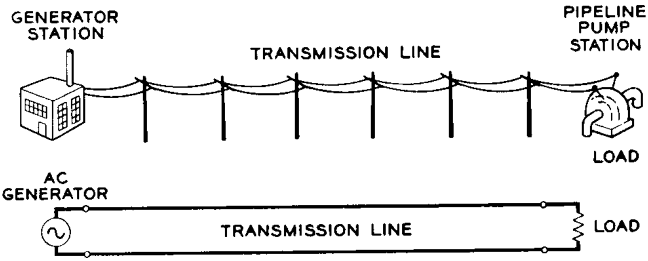


Figure 10. A system for the efficient transmission of power between the generator and the place where it will be used.

The concept of the impedance of a wave medium is perfectly general. It can be applied to media of all kinds. In fact, the term originated in the field of electrical engineering. Electrical engineers use the word to denote the ratio of the ac voltage applied at the input end of a circuit to the ac current which flows in the circuit.

Impedance is a term which can be used to denote how easily a wave may be launched on a particular medium. It is very similar in meaning to resistance, which concerns the ratio of cause to effect in cases where the motion is unidirectional rather than oscillating.

One of the more important concerns of modern power technology is the efficient transmission of power from generator to the place where it will be used. This transmission is done by means of electrical waves on transmission lines. Figure 10 is a simple sketch of such a system.

When you want to efficiently transfer power from a line to a load,* you don't want a lot of reflection to take

* The word "load" is employed to denote the thing at the end of the line which uses the power. A load can be a lamp, a motor, a home appliance, or many such things in parallel.

place at the load end of the line.

It can be shown both theoretically and experimentally that, when a transmission line is delivering power to a load, the delivery is maximum when the impedance of the load is made equal to the impedance of the line. If this condition is fulfilled, all the wave energy traveling along the line is completely absorbed by the load, and there is no reflection whatsoever. The load is then said to be *matched* to the line. If the impedance of the load is different from that of the line, only part of the wave energy will be absorbed by the

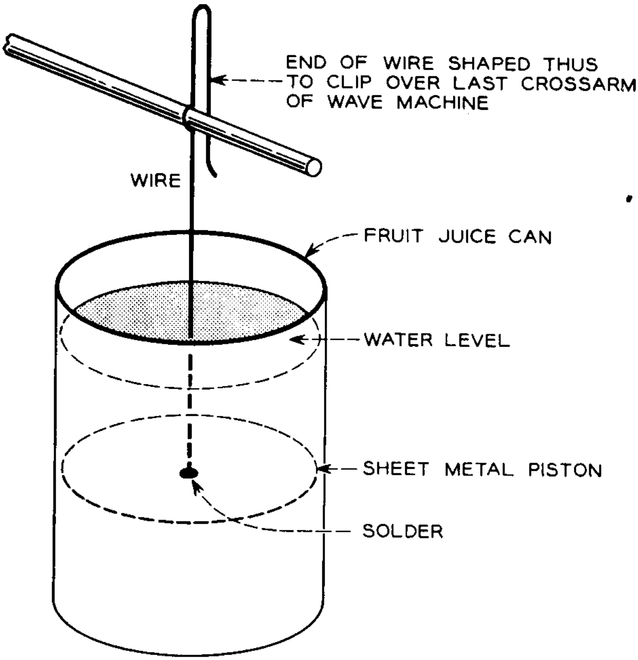


Figure 11. The dash-pot-and-piston arrangement used as a mechanical load for the wave machine.

load, and the rest will be reflected back toward the generator. The amount of energy which is thus lost by reflection increases as the impedance mismatch between load and line becomes greater.

All of these impedance matching and mismatching consequences can be demonstrated beautifully with your wave machine.

To begin, you'll need a mechanical load. This is a simple affair which can be made from a tin can, a sheet of metal, a piece of wire, and a dab of solder. Figure 11 shows how to assemble these parts into what amounts to a dash-pot-and-piston arrangement.

The piston must fit very loosely in the can. There should be no friction between the rim of the piston and the walls of the can. Energy absorption should be due solely to the pumping of water back and forth around the edge of the piston. The piston and wire assembly should be made as light as possible, in order not to tip the wave machine off balance when it is clipped onto the last crossarm. If you pump the piston assembly up and down with your hand you can feel the impedance with which it resists your effort. When clipped on to the last crossarm of your wave machine, it becomes an energy-absorbing load.

The impedance which this load presents to the wave machine depends on how far out it is attached on the last crossarm. There will be some point of attachment along the crossarm where its termination will be equal to the impedance of the wave machine.

To find this point, proceed as follows: Start with the dash pot attached to the last crossarm, about an inch out from the central wire. Manually, launch a single pulse at the input end of the machine and observe approximately what fraction of its original amplitude is reflected at the dash-pot end. Make sure this reflection is right side up. Now move the point where the dash pot is connected out along the last crossarm. Launch another pulse, and you'll see that the amplitude of the reflected fraction of the pulse is smaller than before. The reflection is only partial. Repeat this procedure until you locate the proper point of attachment at which all the pulse energy is absorbed by the dash pot and no reflection at all occurs. The impedance of the load is now matched to that of the machine.

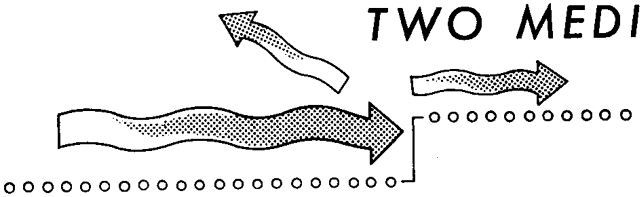
The electrical counterpart of this condition is the one sought after by telephone engineers to minimize bothersome reflections and echoes on long-distance telephone circuits. The acoustic counterpart of this condition is sought after by acoustic engineers in designing sound-absorbing coverings for the walls of rooms which must have no echoes or reverberations.

With the load in this matched position, connect the motor and eccentric drive crank to the input crossarm and send continuous waves down the line. Observe the succession of wave crests marching along and disappearing at the dash-pot end as they deliver all their energy to the dash pot. The absence of reflection means there will be no evidence of standing waves on the machine. You should see only one-way running waves.

Now detach the dash pot and re-attach it near the end of the last crossarm. Out here, the dash pot has a higher impedance than the wave machine and, as you can verify with either continuous waves or hand-launched pulses, partial reflection occurs again. But, this time, the reflected waves are upside-down rather than right side up.

Experiment 11

PARTIAL REFLECTION AT THE BOUNDARY BETWEEN TWO MEDIA



In the last chapter, we saw that the amount of partial reflection of waves at the load-terminated end of a wave medium depends on the impedance mismatch between load and medium. The greater the mismatch, the greater the reflection. Now we're ready to develop the idea that partial reflection also occurs when the output end of one propagation medium is connected to the input end of a second propagation medium having a different impedance. In such a case, the input end of the second medium may be looked upon as a mismatching load at the end of the first medium.

To demonstrate the truth of this generalization, take the long-crossarm wave machine and connect it end to end with the short-crossarm machine. This second machine is built of the same materials and in the same manner as the first machine. The crossarms on the second machine, however, are about half as long as those on the first. The connection can be made by bending up the last quarter inch of the central wires

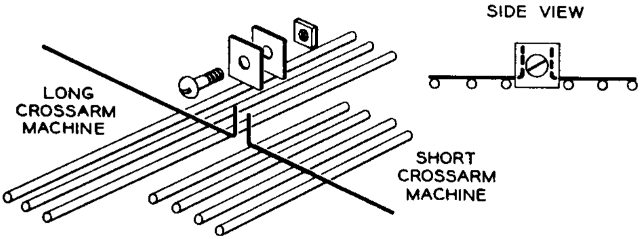


Figure 12. The screw clamp arrangement used to connect the two wave machines together.

of both machines and clamping the turned-up ends with a screw clamp as shown in Figure 12.

Now you have a single transmission medium made up of two sections having different impedances because of their different crossarm lengths. Launch a short pulse on the input end of the long-crossarm machine. Observe that the pulse is partly transmitted across the connection to the second machine and partly reflected from it back to the first machine.

As the transmitted portion of the pulse enters the second machine, it speeds up to a speed appropriate to the second machine and lengthens out accordingly. The reflected portion of the pulse returns right side up, as befits reflection at a lower impedance mismatch. It is diminished in amplitude and has a speed appropriate to the first machine.

In your own experiments, you will be able to make cleaner observations if you first connect the dash pot in a matching position at the output end of the second machine. Doing so will kill the reflections which would otherwise occur there. This makes it easier to follow the reflection and transmission of the main pulse at the connection between the two machines.

Now, instead of using single pulses to investigate the partial reflection at the connection point of the two wave machines, attach the motor and eccentric crank to the input crossarm of the first machine and send out continuous waves. But, before you start the motor, attach the dash pot in a matching position to the last crossarm of the second machine. This will prevent unwanted reflections from hashing up the picture I want you to see. Now start the motor and watch the wave pattern which develops on the first machine. There will be two types of wave: (1) waves going down to the connecting point, and (2) waves of the same period and wave length, but of smaller amplitude, returning in the opposite direction because of the partial reflection at the connecting point. You should, therefore, expect some sort of standing-wave pattern on the first machine. And, indeed, if you look sharply, you will see a standing-wave pattern consisting of a regular series of loops and nodes.

However, the nodes of this pattern are not completely stationary, as they were when we had a totally reflecting termination. In the present case, reflection is only partial. The two wave trains passing through each other on the first machine have different amplitudes, and the pattern you see may properly be called a partial standing-wave pattern.

To determine quantitatively the amount of partial reflection occuring here we must first calculate what is called the standing-wave ratio. This ratio is defined by the following equation.

$$\mathrm{SWR} = \frac{A_{\max}}{A_{\min}}, \qquad (11\text{-}1)$$

where

SWR = standing-wave ratio,

$A_{max.}$ = maximum amplitude of the standing-wave pattern halfway between adjacent nodes, and

$A_{min.}$ = minimum amplitude of the standing-wave pattern at a node.

Now you can calculate the fraction of energy reflected at the boundary between the two machines, through the use of equation 11-2.

$$E_R = \left(\frac{\text{SWR} - 1}{\text{SWR} + 1}\right)^2, \qquad (11\text{-}2)$$

where E_R = fraction of energy reflected.

There is yet another way to determine the amount of energy reflection. The following equation was developed by Lord Rayleigh in work on light waves. However, since light waves, acoustical waves, electrical waves, and mechanical waves are basically alike, in their behavior, equation (11-3) should also be valid for waves on the machines as well.

$$E_R = \left(\frac{S_2 - S_1}{S_2 + S_1}\right)^2. \qquad (11\text{-}3)$$

In this equation,

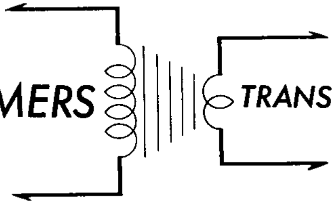
S_1 = wave speed on the large machine in inches per second, and

S_2 = wave speed on the small machine in inches per second.

Try making these measurements and calculations to see how closely the results of equation (11-3) agree with those of equation (11-2).

Experiment 12

TRANSFORMERS



TRANSFORMERS

In modern technology, it is often necessary to transmit wave energy from one propagation medium to another of a different impedance, and in most all cases, partial reflection normally takes place. However, partial reflection is economically wasteful, since the portion of energy being reflected never reaches the other end of the system. The question is, "How can you transmit wave energy from one medium to another, across an impedance discontinuity, *without* suffering reflection losses?"

Transformers provide the solution. A transformer is a device which is inserted into the propagation path at the point where impedance changes abruptly. It has the effect of smoothing over the discontinuity so that reflection is minimized or prevented altogether. In short, transformers allow us to accomplish reflectionless transfer of energy.

The simplest type of transformer is one which, when inserted between two media having different impedances, becomes itself a wave medium with a gradual impedance taper along its length. At its two ends, it has impedances matching the impedances of the media to be joined. Between the ends, the impedance tapers from

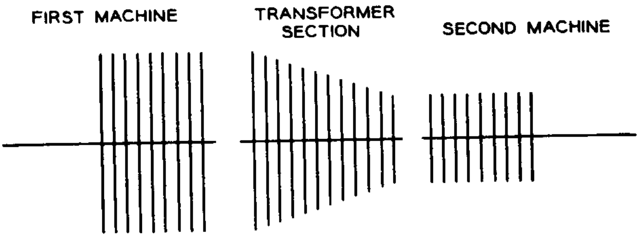


Figure 13. The taper transformer section used to reduce the reflection of waves and pulses.

high to low, or vice versa — depending on which way you are looking at the transformer.

You can easily make a taper transformer section to sandwich in between your two wave machines and see for yourself how its presence reduces the reflection of waves and pulses.

Proceed as suggested in the sketch of Figure 13 with a 10- or 12-inch section of wave medium. Use crossarms of the same diameter, spaced the same distance apart center-to-center along the backbone as on your two wave machines, but with each crossarm shorter by the same amount than the one next to it. The first crossarm of your transformer section should have the same length as the crossarms of your first wave machine; the last crossarm should have the same length as those of your second wave machine. The transformer section may be supported in bearings similar to those used for your two main machines, and may be joined to these machines by the same kind of clamping arrangement previously described. Figure 14 is a photograph of my two wave machines joined by this kind of taper transformer.

Now launch a short, quick pulse on the input end of the first machine. Watch it travel along and transfer across the taper-section transformer onto the second machine. Note the small amount of reflection at the transformer — much less than there would have been without it. The transformer has efficiently smoothed over the impedance discontinuity and permitted a more effective transmission of wave energy to the second machine.

If you learned about electrical transformers in an elementary physics course, they were probably presented to you as devices for stepping up or down voltages applied to their primaries. You learned that if an ideal transformer is used to step down a voltage, it will step up the current in the same ratio.

In analogous fashion, the mechanical taper transformer gives a step down in torque and a step up in angular velocity and angular displacement by the same factor.

There is another type of transformer that we may use in experimenting with the wave machines. This device is known as a quarter-wave transformer. It consists of two reflecting steps a quarter of a wave

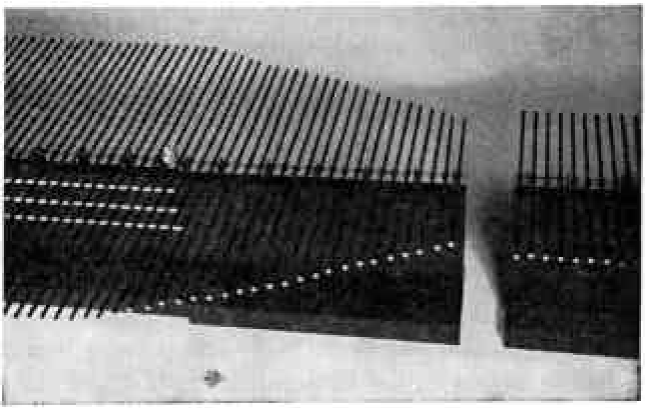


Figure 14. The taper transformer section sandwiched between the two wave machines.

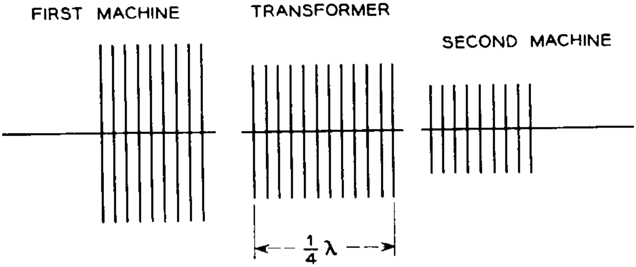


Figure 15. The quarter-wave transformer section for use with the two wave machines.

length apart such that the partial reflection from the second step cancels that from the first, giving no net reflection at all. Theory tells us that the impedance of the transformer section between the two steps should be the geometric mean of the impedances of the two media you are connecting to the transformer, and that the two steps should, indeed, be a quarter wave length apart.

A quarter-wave transformer section for use with your two wave machines would look something like the one sketched in Figure 15. To make it, you will have to compute how long the section should be and, also, how long to make the crossarms. I'm going to let you compute these for yourself.

If your two wave machines and the proposed transformer have central wires of equal diameter and material. Then, the impedance of each section will be proportional to the square root of the rotational inertia of the crossarm system per unit length, [equation (4-3)]. If you know the frequency of your wave generator and the speed of the waves in your transformer section [these can be computed from equation (4-1)], you can determine how long the section should be made in order to have a length of one quarter wave length. The quarter wave length, by the way, refers to a quarter wave length *in the transformer medium*, not in either of the two machines.

After having built your transformer section according to the above considerations and mounted it in bearings, sandwich it in between your two wave machines and see how effective it is in cutting down reflection losses. To do so, measure the SWR on the first wave machine, first with the transformer section in place, and then with the two machines directly connected without the transformer. Calculate and compare the fraction of energy reflected in both cases. In these measurements, the dash pot should be attached to the output end of the second machine in a matching position.

You can imagine, knowing how a quarter wave length transformer operates by mutual cancellation of the two reflections, that such a transformer will not work very well for single waves and pulses. You can verify this conclusion by experiment. A quarter wave length transformer is effective only with continuous waves. Moreover, it is effective only with continuous waves at and near the particular frequency for which the transformer was designed. The taper transformer, on the other hand, works fairly well with single waves and pulses, as well as with continuous waves. And it does so over a broad range of frequencies. It is subject only to the limitation that the half wave length of the waves must be shorter than the length of the taper section.

SUMMING UP

From examples cited in the experiments, you should once again be impressed with the way so many apparently unrelated physical phenomena turn out to be quite similar after all — once we have stripped away their disguises and revealed their basic essences.

The important thing to note is that waves of *all kinds* behave fundamentally alike as they propagate, reflect, superpose, interfere, and go through their various other paces. Be they mechanical, acoustical, electrical, thermal, optical, or electromagnetic waves, they are basically sisters under the skin. If we learn to understand one, we can understand all.

It pays to keep in mind, when undertaking the study of any new discipline, that at first sight Nature often appears arbitrary and capricious. A closer look, however, will usually reveal basic laws which govern her behavior — laws which are the same in physics, chemistry, biology, geology, astronomy, oceanography, and all the other branches of natural science. Contemplation of these basic laws should convince any skeptic of the tremendous beauty, unity, and universality of Nature, however complicated she may seem superficially to be.

If at the end of your reading of this book I were to ask you what you have learned from doing so, and if you were to reply, "I learned some interesting things about the behavior of waves," I would feel that your experience had brought you some small value for the effort you have expended. But if I were to press you, "Is that all you learned?," and if you were to reply, "Yes, I guess so," I would be terribly disappointed and would wonder whether you were a superficial reader or I an ineffectual writer.

The teaching of this book, if teaching there is at all, lies not alone in the information which it presents, but also in the point of view which it invites you to share. If you were to reply, "It taught me to be more alert for the common features which often tie together seemingly unrelated things", or, "It taught me that nature is basically simple and consistent to the extent that in facing an apparently complicated phenomenon, I may look first for simple answers suggested by things I already know", then I would be certain that this book had really scored.

Has it?

DR. JOHN N. SHIVE

Physicist John Shive has long been fascinated by similarities in various natural phenomena. In this book he describes these likenesses in the behavior of waves of many kinds. His words strikingly reveal the true simplicity of nature's laws.

Dr. Shive received a Ph.D. in physics from Johns Hopkins University in his native Baltimore in 1939. He then joined Bell Telephone Laboratories where he has done research and development work on semiconductors. The phototransistor is one of his major inventions.

He is a leading author on scientific topics, particularly semiconductors.

Dr. Shive is now Director of Education and Training at Bell Laboratories.

Prepared by Bell Telephone Laboratories for educational use.

SHIVE

SIMILARITIES IN WAVE BEHAVIOR