THE TEACHER'S EDITION

SIMILARITIES IN WAVE BEHAVIOR

BY DR. JOHN N. SHIVE
DIRECTOR OF EDUCATION & TRAINING
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If you wish to construct your own wave motion machine, you can do so from the information provided in Chapter 2 of this book. Or you can purchase one or more machines from:

Science Department
Allegri-Tech, Inc.
141 River Road
Nutley 10, N. J.

The prices are:

Machine, completely assembled, with two fiber carrying cases—$124.00

Machine, completely assembled, without carrying cases—$84.00

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FOREWORD

This book has two purposes.

The first is to set forth the more elementary concepts and facts in the particular area of knowledge known as wave behavior.

The second is to teach by example, implication, and outright explication some of the more lost-sight-of ideas about science and its approaches, ideas for which the subject of wave behavior serves as an unusually suitable vehicle.

And of these two purposes, I believe the second to be equally as important as the first.

To many people science is confused with knowledge. Almost any and every array of facts is labeled "scientific". The bigger the welter of information we can amass about the universe and everything that's in it, the more "scientific" we imagine ourselves to be. When the kiddies in the third grade of the local school begin taking nature study, the parents proudly imagine that their youngsters are starting science. High school science fairs are loaded with rock collections, technology demonstrations, and model-building exhibits that more properly belong in hobby shows. The reading public is constantly drenched with articles and advertising which make a sacred cow out of science, and then misuse it to "prove" almost anything the writers want to prove. We Americans are terribly confused about what science really is, what its methods are, what its limitations and particular fields of application are, and what its real position is in the scheme of human endeavors.

Science is one of the several formalized disciplines through which man seeks satisfaction for his curiosity, answers to his questions, and solutions for his problems. It stands on an equal footing with religion, ethics, philosophy, and the arts as one of the foundation stones of human culture. As with these other disciplines, there are particular areas where science may effectively be applied, and there are limits outside of which it is virtually worthless. You wouldn't turn to science for an answer to the question of what love is, any more than you would look to religion for a solution to a problem in geometry.

Science is not knowledge; it is a method. It is a method for systematizing existing knowledge and for developing new knowledge through the techniques of observation, hypothesization, experimentation, and analytical operations with data. Among these techniques, too, are the discovery and establishment of relationships between the variables of nature, relationships which define the dependence of one thing upon another and lead us to the understanding of our environment which is prerequisite to being able to control it for our own purposes. Upon this understanding do our engineering and technology depend.

One of the most powerful techniques for systematizing our knowledge is to look for ways of generalizing the results of our observations. When we observe that stones roll always downhill, that charge flows always along a conductor from a point of higher potential to a point of lower potential, that heat flows always from a region of high temperature to a region of lower temperature, and that an excited atom sooner or later radiates or otherwise gets rid of its excess energy, we

generalize, "The natural readjustment of an unstable system will be in the direction of decreasing its free energy". When we observe that momentum is conserved in all experiments involving bodies in motion without external friction, we gather all these observations together into a generalization now known as the law of the conservation of momentum. Such a generalization, arising from our observations of behavior in a number of similar situations and from the assumption that nature is and will continue to be consistent, permits us to predict behavior in similar situations yet to be encountered.

Wave behavior is an excellent subject with which to exemplify these features of the scientific method. It cuts across many fields of physics and engineering. In all these fields we find waves of various kinds doing similar things with a predictability which strengthens our confidence in the consistency and universality of nature's laws and in our understanding of them.

And now let's get on with it.

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Similarities in Wave Behavior

Admonition to the reader: if you haven't read the Foreword, go back and do it. Right now!

CHAPTER ONE

Preliminary Ideas

Waves of one kind or another may be encountered everywhere. The lovely colors of a summer sky, the sound of a passing auto, the restless surface of the sea, the gentle undulation of windblown wheat stalks, and even the flapping flag or clothesline in a neighbor's yard are all familiar manifestations of wave phenomena.

Less perceptible to our senses, perhaps, but of tremendous importance to our lives, are radio and TV waves, X-ray waves, ultra-violet and infrared waves, electrical waves on power and telephone lines, and waves of electrochemical reaction along nerve fibers.

The involvement of waves in Nature's vast scheme of things gives us ample reason to learn more about them. It is my earnest hope that you will find such study fascinating for its own sake, as well as instructive for its scientific and technological value.

A Simple Definition

Before we get any deeper into the subject, let's pause long enough to consider a definition. *A wave is a traveling disturbance of a medium from its normal condition.* Now let's examine this definition piece by piece.

The medium is the thing in which or along which the wave travels. Examples of such media are water

TABLE I-1

Summary of Information About Waves of Various Kinds

Category	Type	Medium	Nature of the Disturbance	Typical Wave-Length Ranges	How Produced
Mechanical	Transverse	stretched strings and cables, drumheads, solids (as for earthquake waves)	lateral vibration crosswise to direction of travel	few inches to a few yards	vibrating objects in contact with the medium
	Longitudinal (sound)	air and other gases, liquids and solids, freight-train couplings	vibration back and forth along the direction of travel	0.0001 cm to several hundred yards	
	Torsional	wires, rods, bars	twisting back and forth around direction of travel	inches to feet	oscillating source of twist
	Liquid Surface	water and liquid surfaces interfaces between liquid layers	combination of transverse and longitudinal vibrations (See Text)	millimeters to miles	wind, moving ships, dropping objects, liquid layers in relative motion
Electromagnetic	All electromagnetic waves are transverse	empty space: since all matter is composed mostly of empty spaces except for the nuclei and orbital electrons of the atoms. electromagnetic waves travel in all solids, liquids. and gases except those whose nuclear, electronic, or atomic arrangements produce absorption in various wave-length ranges	presence of oscillating electric and magnetic fields	*Radio*, few hundred yards to a few miles *TV*, few feet to a few yards *Microwave*, few mm to a few inches *Infrared*, 8000 Angstroms to 0.1 mm *Visible Light*, 4000 to 8000 Angstroms *Ultra-violet*, 100 to 4000 Angstroms *X-Ray*, 1 to 100 Angstroms γ-*Ray*, 0.001 to 1 Angstrom	electric charges oscillating back and forth in antennas and similar structures electronic oscillations in outer orbits of atoms and molecules electronic oscillations in inner orbits of atoms nuclear oscillations
Electrical	Longitudinal	power transmission lines, telephone wires	oscillating electric fields and currents along the wires	few inches to a few miles	ac power generators, microphones
Thermal	Longitudinal	solids, liquids and gases possessing thermal conductivity	fluctuations of temperature and heat flow	millimeters to yards	source of fluctuating temperature

surfaces, air, glass, stretched strings, power lines, wave guides, even empty space.

The normal condition of a water surface is to lie smooth and level. However, we can alter this condition by tossing a stone into the water. This simple act creates a disturbance in the form of a series of concentric crests and troughs. The disturbance then travels outward from the center where the stone fell. It is important to note that the water doesn't travel with the waves; only the condition of disturbance — that is, *the wave, itself*, moves along. The water remains substantially where it was, with various portions of its surface merely bobbing up and down as the waves go by.

The normal condition for air in a room is to be at a uniform pressure everywhere. However, if someone strikes a bell in the room, the vibrations of the bell are communicated to the air layers adjacent to it. These layers, in turn, transmit the vibrations to the next farther layers, and so on. The air vibrations are transmitted outward from the bell with the speed of sound, causing fluctuations of pressure and density in the successive air layers through which the sound passes. Such fluctuations comprise the disturbance of the medium (air) from its normal condition. Here, again, the air doesn't travel away from the bell. The individual air molecules are merely jostled back and forth about the positions they occupied before the sound waves arrived. Only the pressure fluctuations are actually traveling.

From the foregoing examples, it should be evident that waves in various media are caused by various disturbance-producing mechanisms. A few such mechanisms are winds, moving ships, vocal cords, glowing gas discharges, incandescent lamps, radio and TV

antennas, fluctuating temperature sources, and suchlike.

Information about various kinds of waves is summarized in Table 1-1. The table compares propagation media, typical wave lengths, and the nature of some disturbances and disturbance-producing sources. In it are revealed some startling extremes. Electromagnetic waves, for example, can have wave lengths as short as a thousandth of an Angstrom unit* or as long as several miles. It is almost beyond belief that waves differing in length by thirteen powers of ten (ten million million times) act so much alike in many aspects of their behavior. Nevertheless, they do.

It will be the purpose of this book to describe a number of wave phenomena gathered from various fields of physics and engineering. Some of these phenomena will be as familiar as the fingers of your hand. Others you may never have encountered before. Yet all of them will be so logically related as to reveal the beautiful simplicity of nature. And all have been chosen so they can be demonstrated with mechanical apparatus which you can obtain or build for yourself. Every feature of wave behavior demonstrated by this apparatus has its counterpart in the behavior of electrical waves on transmission lines, sound waves in gases, light waves in optical media, and waves in liquids and solids. It is hoped that learning about waves with the aid of mechanical demonstrations which you can see and operate yourself will assist you in the understanding of waves of all kinds. The hope is a valid one, since all waves exhibit behavior features in common.

Studying these relationships affords an excellent introduction to the scientific approach. The scientist

* One angstrom unit is 10^{-8} cm.

includes among his responsibilities (1) searching for common features in the phenomena under his observation, (2) associating these common features through generalizations called hypotheses or laws, and (3) subsequently applying these generalizations to the solution of related problems. The processes of observation, analysis, generalization, and application exemplify the practice of science at its best.

The broad subject of wave phenomena, with its ramifications relevant to so many different fields of physics, offers an excellent opportunity for demonstrating to science students how these operations work.

CHAPTER TWO

How to Build a Wave Machine

NOTE: This chapter may be omitted by anyone who has acquired a wave machine already assembled or in kit form. Such kits contain complete directions for assembly. The following was written for those who wish to cut down expenses by making their own wave machines from materials begged, borrowed, stolen, or even, in the last extremity, purchased.

The wave machine consists of a backbone-and-cross-piece structure supported in bearings which permit the structure to oscillate. Figure 2-1 shows a photograph of the machine completely assembled with a single wave traveling along it. Figure 2-2 is an exploded-view drawing of the machine showing the various parts separately and indicating how these parts are put together in the completed assembly.

In my machine the central backbone is a 3-foot long

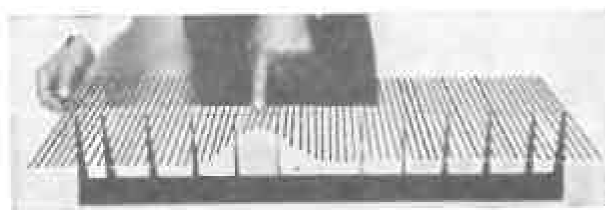

Figure 2-1. A torsion wave is traveling along this assembled wave machine.

piece of 0.042-inch steel drill rod. The crosspieces are 18-inch lengths of 0.150-inch steel drill rod. These are soldered to the backbone exactly at their centers, parallel with each other and at right angles to the backbone. In the machine pictured in Figure 2-1, there are 70 of these crosspieces evenly spaced about ½ inch apart along the backbone. Ordinary rosin-core soft solder was used to make the attachments.

The necessary metal parts may be obtained in a hardware store or metal shop. Doubtless, piano steel wire, which is very springy, would be better for the

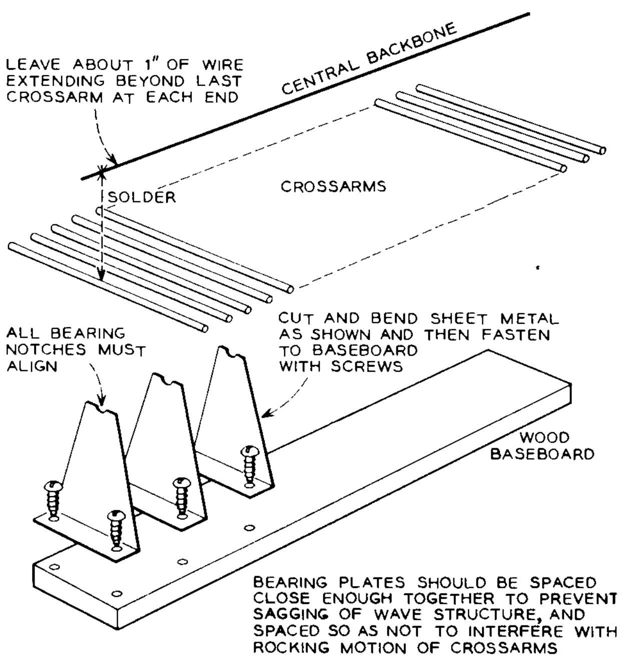

Figure 2-2. This exploded-view sketch shows the arrangement of the parts of the wave machine.

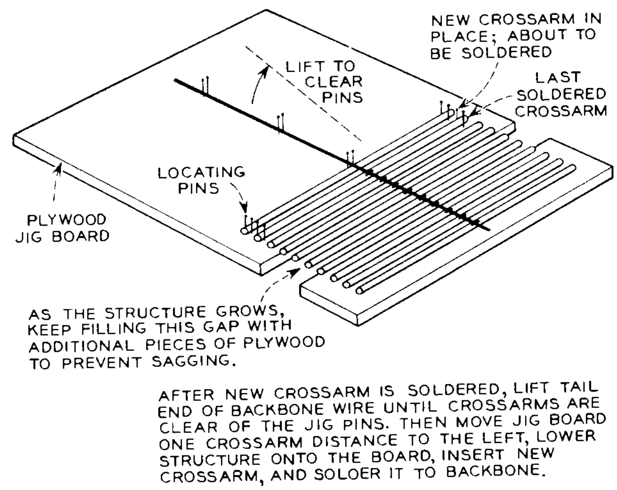

Figure 2-3. Use of a jig will help you to do an even job of soldering the crossarms onto the backbone.

central backbone than drill rod, but it can't be easily soldered with soft solder.

There is nothing critical about the lengths or diameters of either the central backbone or the crosspieces. The larger the diameter of the central backbone, the stiffer it will be to twisting, and the faster the waves will travel along the machine. The larger the diameters and longer the lengths of the crosspieces, the more rotational inertia the crosspiece structure will have, and the slower the wave will travel along the machine. Crosspieces of square or rectangular cross section can be used if desired, and will do just as nicely as the cylindrical rods described.

A Helpful Jig

A neat job of soldering the crossarms to the central backbone can be done if a jig is used to insure even spacing. Such a jig (shown in Figure 2-3) is made by arranging small wire nails in the proper pattern on a plywood baseboard. One set of nails holds both the central backbone and the last soldered crossarm wire. Another set holds the next crossarm in proper relation to the rest of the structure while the solder connection is made. After each soldered connection is completed, the jig is moved one space farther along to set the stage for the next repetitive operation.

My friend, C. E. Briggs, has devised what may turn out to be an even simpler arrangement for holding the crossarms and backbone in proper relation to each other while solder connections are made. He took a 3-foot board, 6 or 8 inches wide, and cut grooves across the top surface, parallel to each other and perpendicular to the length of the board. The grooves were $\frac{1}{2}$ inch apart and deep enough to hold the crossarms securely in place while the central wire was laid along the center

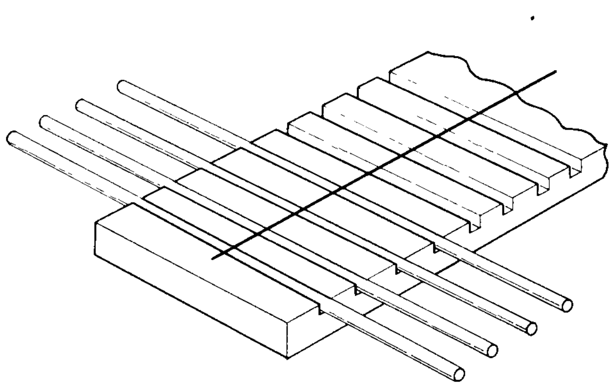

Figure 2-4. Mr. Briggs' slotted board makes an excellent soldering holder for the crossarms.

line on top of the array and soldered to each crossarm. Mr. Briggs made quick work of slotting the board by using a jig on the miter gauge of his power saw. Figure 2-4 shows the board with a few of the crossarms placed for soldering.

SPECIAL NOTE for the advanced reader who wishes to build a more elegant machine

A wave machine built according to these directions will give you hours and hours of satisfaction in its operation. However, an even better machine can be built, and a few more words about it may decide you to build it. It requires only a little extra effort.

When the machine described above is mounted in its bearings, the entire crossarm structure has a natural period of oscillation when disturbed by displacement

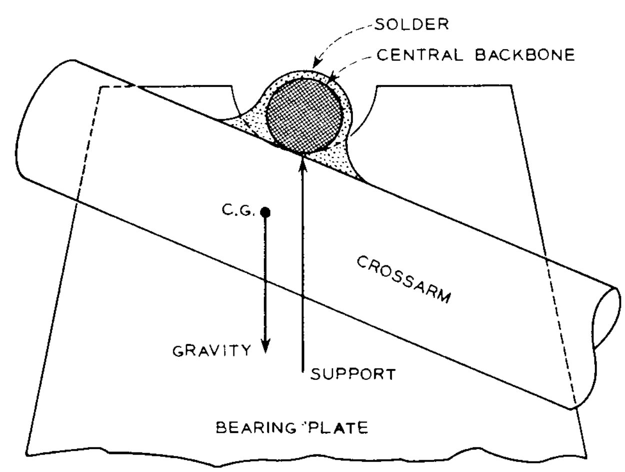

Figure 2-5. When the crossarm is rocked a restoring couple is produced, causing the crossarm to oscillate naturally.

from its normal horizontal plane. Each crossarm acts like the beam of a chemical balance. As can be seen in Figure 2-5, whenever the crossarm is rocked from its horizontal position, the center of gravity is moved out of the vertical line through the point of support. A restoring couple is brought into play which causes the crossarm to oscillate back and forth like a swing. The period of this oscillation may be three or four seconds. Every time a single wave is launched on the machine, a certain amount of this natural oscillation is generated also. If the period of the natural oscillation is close to the period of the wave (or waves) being investigated, your observations may be fuzzy and difficult to interpret.

To avoid this difficulty you can make a machine whose natural period is so long — eight to ten seconds, say — that there will be little likelihood that the structure's natural period will ever become a significant distraction.

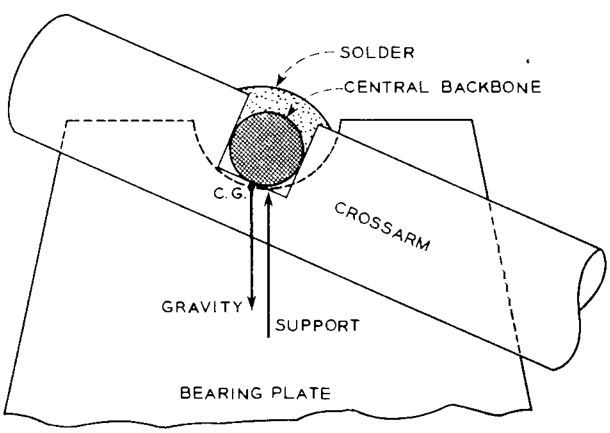

Figure 2-6. With a slotted crossarm the restoring couple is smaller and the natural period is longer.

To obtain the longer natural period, cut a slot in the exact center of each crossarm before assembly. Each cut should be wide enough to accommodate the diameter of the central wire and deep enough to go just halfway through the crossarm. The central wire is then sunk into the slot before soldering. The center of gravity, as shown in Figure 2-6, will now move only slightly away from the vertical line of support when the crossarm structure is rocked. The restoring couple is smaller than with the unslotted construction, and the natural period of oscillation of the crossarm structure is longer and less likely to be bothersome.

Other Considerations: Mounting and Finishing

If you have any difficulty obtaining metal parts for the crosspieces, you might want to experiment with wooden dowel rods. These can be fastened to the central backbone with some of the iron glue cements available in hobby shops. If waves travel too fast on such a machine, you can always slow them down by attaching equal additional masses to both ends of all crosspieces. The more mass, the lower the wave speed. I'm afraid there's no escape from making the central backbone of metal. Wood, plastic, stiff rubber tubing, and the like, may be springy enough, but they all exhibit internal hysteresis which would prevent the waves from traveling very far without being quickly attenuated because of energy absorption in the central backbone.

The wave structure itself rests in U-shaped notches in the top edges of the sheet metal bearing plates which are secured to the wooden baseboard as shown in Figure

2-2. The U-shaped slots should have a radius several times that of the central backbone. This allows the twisting central backbone to roll freely in the bearings when waves travel along the machine. The space between the bearing plates is not critical; it should merely be small enough so the wave structure doesn't sag appreciably between bearings. The bearing plates should support the central backbone without coming into contact with any of the crossarms or in any way interfering with their rocking motion.

After you have completed the fabrication of the wave structure in the jig, you will have to transfer

Figure 2-7. To mount the machine lift the wave structures and lower it into the notches of the bearing plates.

it to the bearings of the base and mount. To do this with a minimum of damage, grasp the two ends of the machine where the central backbone wire is soldered to the first and last crosspieces. Then, without twisting or bending the central wire, lift the structure, simultaneously pulling outward on both ends to keep the medium from sagging beyond the elastic limit of the central wire. Lower the structure into the U-shaped notches in the bearing plates (Figure 2-7) making sure the ends come out where they should. If you have properly located the bearing plates, they should not interfere with the motion of the crossarms.

Now you are ready to start a wave and watch it go. Gently pump the end of the first crossarm up and down once. The resulting wave will detach itself from your hand, run down to the other end of the machine, reflect and return, repeating the cycle several times before dying out. What, indeed, hath God wrought!

Dressing Up the Machine

You have the machine working now, but you may want to improve its appearance a little. If so, a coat of push-button spray paint can be applied to both the wave structure and the mounting in a few minutes. If the crossarm structure doesn't lie exactly horizontal in its bearings, a little extra paint applied to the higher side will balance it up. If you plan to demonstrate your machine to an audience, a dab of white paint on the end of each crossarm will make the waves easier to follow.

A really dramatic effect can be produced by painting the machine black and dabbing a bit of fluorescent white paint on the end of each crossarm. It can then be exhibited in a dark room with an ultra-violet lamp.

If you wish to perform all the demonstrations described in this book, it will be necessary to build a second wave machine and a few other miscellaneous items as well. Most of these will be described in the chapters in which they first appear.

The second wave machine is built just like the first except that the crossarms are made half as long. The angular rotational inertia of the crossarm system will thus be less than that of the first machine, with a consequent increase in wave speed of about three times.

The second machine will require a mounting base similar to the one used for the first machine. However, since the weight of the crossarm system is less, the bearing plates can be spaced about half again as far apart.

CHAPTER THREE

Some Simple Wave Demonstrations

NOTE: This chapter and its exercises may be bypassed by persons familiar with the more elementary features of wave behavior.

Now that you have a wave machine, you are probably itching to do something with it. In this chapter, I will cover some features of elementary wave behavior which can easily be demonstrated with your machine. Please keep in mind that, while the waves you produce and watch are mechanical, the principles discussed and demonstrated are equally valid for waves of all kinds. Nature exhibits a simplicity and universality which are beautifully borne out in the aspects of wave behavior we shall study.

First of all, start another single wave traveling along your machine. Observe that the twisting springiness of the central wire is all-important to the propagation of waves on the machine. When you pump the end of the first crossarm up and down, what you are really doing is twisting and untwisting the end of the central wire to which the crossarm is attached. The twisting and untwisting are then transferred by the springiness of the central wire to successive sections farther along its length. The wave is thus actually a wave of twisting and untwisting of the central wire, made perceptible to you by the action of the ends of the crossarms which bob up and down as the twist wave travels along the wire. The ends of the crossarms exhibit a transverse type of displacement, moving up and down for short distances while the wave itself travels horizontally along the structure.

Start another wave and ask yourself if it fulfills the wave definition which we considered in Chapter One. *A wave is a moving disturbance of a medium from its normal condition.* In this case the central wire and crossarm structure comprise the wave medium. Its normal condition is to lie horizontal and motionless in its bearings. When we launch a disturbance on the structure in the form of a transverse displacement of the crossarms, this disturbance propagates itself along the machine and produces something which fits every condition of the definition. It is, therefore, quite properly called a wave.

Note that the portions of the machine, both ahead of the wave and behind it, are essentially at rest. The wave itself is thus a purely local affair which travels along the medium at its own good speed, this speed being determined by the physical properties of the medium.

Pulses and Continuous Waves

In discussing wave behavior, we shall often make the distinction between single waves, or *pulses*, and continuous waves, which are long trains of single waves periodically repeated at equal intervals.

You can start a train of continuous waves on the wave machine by repeatedly pumping the end of the first crossarm up and down. If you do not already have a prepared wave-machine kit with a wave generator motor, you may wish to make such a generator on your own. You can do this by taking an erector-set or elec-

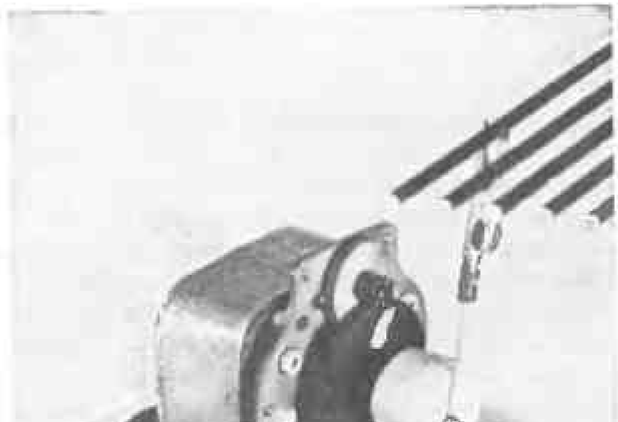

Figure 3-1 A geared-down motor generates continuous waves.

tric-clock motor, gearing it down to a speed of one or two revolutions per second, attaching an eccentric crank to the shaft of the speed reducer, and then driving the first crossarm of the wave machine by means of the crank rod. The amplitude of the waves can be increased by moving the crank rod clip farther along the crossarm toward the central wire. Figure 3-1 is a photograph of a synchronous midget motor with a speed-reducing gear train generating continuous waves.

Damping

Perhaps one of the first things you notice about the waves on your machine is that they don't keep going forever. Their amplitude decreases steadily until, after a few reflections between the ends of the machine, they disappear altogether.

This decrease in amplitude, or *damping*, is due to friction. In the mechanical wave machine there are three sources of friction, each of which independently produces its share of damping. These are: (1) air friction operating on the moving crossarms, (2) rolling friction of the central wire twisting back and forth in its bearings, and (3) internal hysteresis friction in the central wire itself. Each of these sources of friction is continually abstracting energy from the wave and converting it to heat. The wave, as a result, diminishes in size and ultimately dies out.

The damping of waves of all kinds is a fairly general phenomenon of nature. Sound waves, for example, become fainter as we get farther away from their source. Most of this decrease is not true damping, but rather the familiar inverse-square law falloff due to the increasing area which the wave front has to cover as it spreads outward spherically from the source. At the same time, however, there is true energy absorption by the air the sound passes through. As the air molecules jostle back and forth, some of the organized motion due to the sound waves is converted into disorganized motion, which results in heating of the air. Moreover, some of the compressional heating occurring in those layers of air which at any instant are under compression is continually leaking away to adjacent layers of air which at the same instant are under rarefaction and hence are cooler. This heat conduction tends to smear out the pressure fluctuations and consequently results in damping of the wave intensity. These two processes of true damping operate

over and above the ordinary inverse-square law decrease of intensity with distance from the source. They may thus be regarded as instances of acoustic friction.

Light waves, too, are absorbed in greater or lesser degree by almost everything they pass through, including objects which we ordinarily regard as transparent, such as air, water and glass. Daylight cannot penetrate through more than a few hundred feet of even the clearest water. And the setting sun appears red because the green and blue components of sunlight are being selectively absorbed by the increased thickness of atmospheric air through which the sun must be viewed when it is near the horizon.

Electric waves on telephone wires and transmission lines are similarly damped because of the electrical resistance of the wires. As the waves travel along, the electrons in the wires surge back and forth. In this movement, they encounter frictional resistance which dissipates the energy of the wave in the form of heat. Without help, the electric waves which carry a telephone conversation along telephone lines die out within 15 or 20 miles. Long-distance lines must be specially engineered to get around this difficulty. On such lines the damping is compensated for along the way by amplifiers which are inserted into the circuit every few miles to restore the original intensity of the waves.

The only known case of wave propagation in which there is no damping whatsoever is the propagation of electromagnetic waves in absolutely empty space. Because this is so, light from distant nebulae can be observed on earth after traveling through interstellar distances of 10^{19} miles, undimmed except by the normal inverse-square law decrease of intensity.

For the sake of contrast, try imagining what things would be like if the space between the earth and the sun, a mere 10^8 mile hop-skip-and-jump away, were filled with clear air of normal density. We would never see the sun at all. Its light would never reach us because of the absorption in all that air!

Waves as Carriers of Energy

In the last section we talked about the frictional processes that cause wave damping by abstracting energy from waves of various kinds. That waves transport energy can be demonstrated with your wave machine if you first make an auxiliary device like the one shown in Figure 3-2. This is simply a device for lifting a weight by means of a ratchet arrangement which is operated by the up-and-down motion of the last crossarm on the wave machine as continuous waves arrive from the generator. Figure 3-3 is a drawing of this apparatus. Enough detail is shown to enable you to make one like it.

The weight is raised a short distance on each upstroke of the last crossarm. The altitude thus gained is held by the upper ratchet while the crossarm, with its ratchet spring wire, goes down for another lifting stroke. A little experimentation will show you how far out on the crossarm the weight-lifting attachment should be engaged. Engagement too far out toward the end will stall the wave machine; engagement too far in toward the central wire will result in too slow a lifting of the weight.

That waves are carriers of energy can be deduced by philosophical argument as well as proven by mechanical demonstration. To get a wave started on the machine one must displace the first crossarm. Such displacement requires that a force be exerted against the

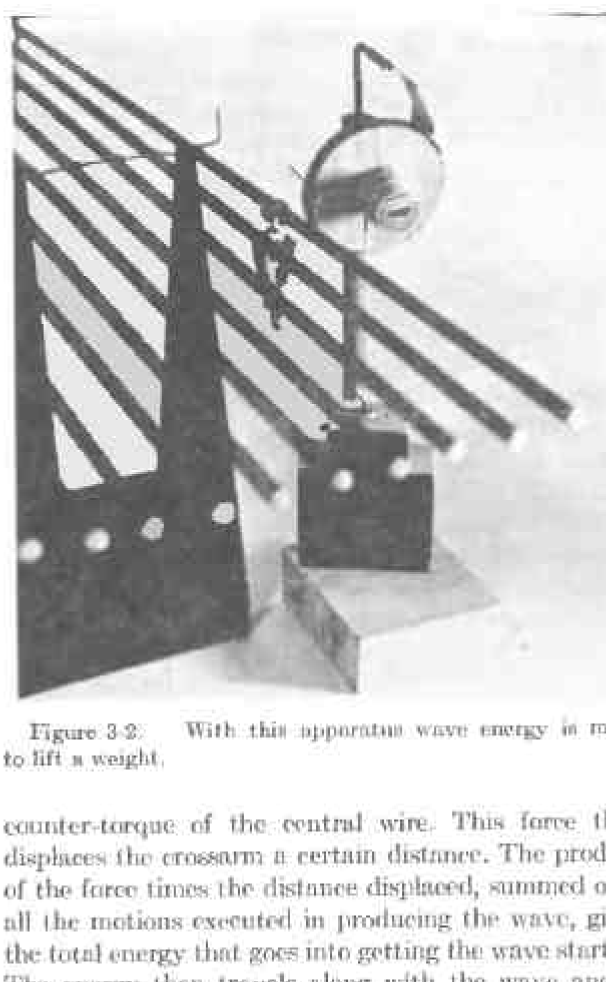

Figure 3-2: With this apparatus wave energy is made to lift a weight.

counter-torque of the central wire. This force then displaces the crossarm a certain distance. The product of the force times the distance displaced, summed over all the motions executed in producing the wave, gives the total energy that goes into getting the wave started. The energy then travels along with the wave and is divided between the potential energy of distortion of the medium and the kinetic energy of those parts of it which are in motion at any instant. The energy of the

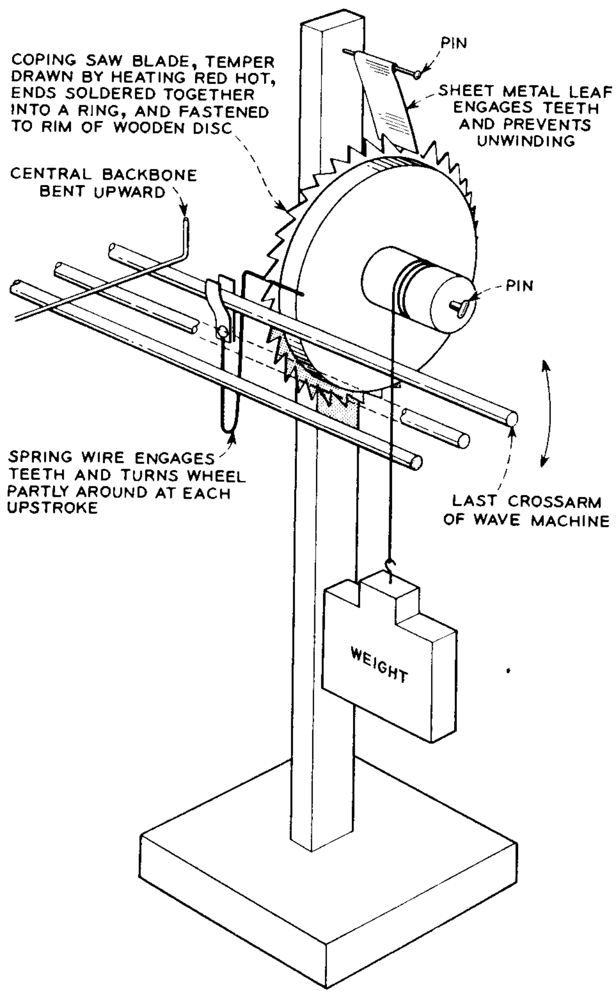

Figure 3-3. You can make one, too. (See text.)

wave can be delivered at the other end of the medium, where it can do useful work, as is demonstrated by the weight-lifting arrangement.

While thinking of mechanical waves traveling along wave machines, you should also be thinking of sound waves traveling through air, electrical waves traveling along wires, electromagnetic waves launched into space from radio and TV broadcast antennas, and water waves on the surface of the sea. All these waves require energy to produce them, and they carry this energy with them as they travel along.

The concept of energy transportation from one place to another by means of waves traveling along wave media underlies many of the things we do in modern technology. We generate electric energy at one place and transport it by means of power transmission lines to other places where it is to be used. We transport information signals around the country with telephone systems and with radio and TV wave broadcasts. Energy is transferred from one place to another in machines by oscillating mechanical linkages.

Relationship Between Amplitude and Energy

You may, some time or other, have heard or read that the energy of a wave is proportional to the square of its amplitude. Simple philosophical proof of this statement can be given for mechanical waves and extended by analogy to waves of other kinds.

Consider the case of a transverse wave medium. The energy carried along by the wave is the same as the energy you expend in getting it started to begin with. A wave having a certain wave length and certain amplitude has also a definite amount of energy. To launch a wave having *twice* this amplitude, you must

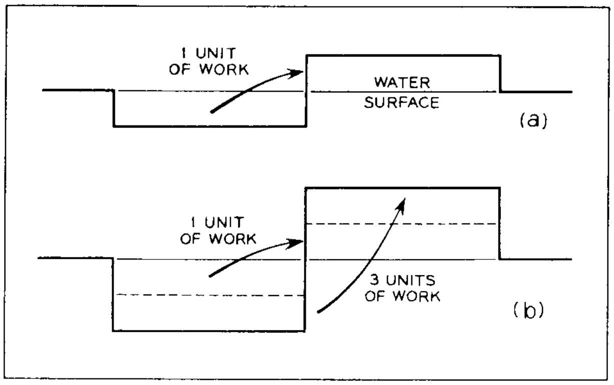

Figure 3-4. To produce a wave having *twice* the amplitude of a smaller wave requires *four* times as much work.

exert *twice* the transverse force if the medium obeys Hooke's law. Since the work done is twice the force times twice the displacement produced, the work is 4 times as great as for the smaller wave. Similarly, to produce a wave having 3 times the amplitude, you must exert 3 times the transverse force, and consequently endow the wave with 9 times the energy.

A good graphical proof that the energy of a wave is proportional to the square of its amplitude may be developed as follows. Think about a water wave. Such a wave is built up by taking water away from the trough and piling it up on the crest. Work must thus be expended to raise this water against the force of gravity. To produce the rectangular wave shown in Figure 3-4(a), suppose that unit mass of water must be lifted unit distance, the resulting wave consisting of a trough and a crest representing one unit of work. To make a wave twice as high and with a trough twice as deep, a second unit mass of water must be lifted 3 units of

distance and piled on top of the first, Figure 3-4(b). To do this requires 3 units of work which, added to the original 1 unit, gives a total of 4 units of work to make a wave having twice the amplitude. By projecting this same argument one step farther, satisfy yourself that a wave 3 times as high requires 9 times as much work to produce. This argument does not lose any of its validity if we round off the corners of the rectangular wave and give it the more familiar sinusoidal shape instead.

Instances of the dependence of the wave energy on the square of the wave amplitude are found in other wave systems as well. The power available from electrical waves traveling along a transmission line is proportional to the square of the wave voltage, since doubling the voltage results in a doubling of the current also, and hence in a quadrupling of the power delivery. Similarly, the intensity (power) of a beam of light is proportional to the square of the amplitude of the oscillating electric field in the beam, and the power in a sound beam is proportional to the square of the amplitude of the pressure fluctuation.

Wave Speed

The speed with which a wave should travel on your wave machine can be calculated from a theoretically derived formula. It can also be determined by direct measurement with a meter stick and a watch. Shall we see how closely these two methods of speed determination agree?

By means of a derivation based on considerations of the torsional springiness of the central wire and the rotational inertia of the crossarm system, it can be shown that the wave speed on the wave machine should be:

$$s = \sqrt{\frac{\tau}{I}}, \qquad (3\text{-}1)$$

where τ is the torsion constant of the central wire (that is, the torque required to produce one radian of twist in unit length of the wire), and I is the rotational moment of inertia of the crossarm system per unit length along the machine.

You can measure τ for your wave machine as follows. Clamp the end of the tenth crossarm of the wave machine so that it can't move. Then hang a small mass (m grams) from the end of the first crossarm so as to produce a 10 or 15 degree twist between the weighted crossarm and the clamped crossarm. Measure the angle of twist, θ, and convert it to radians. Now measure the distance l in centimeters between the centers of the weighted and clamped crossarms, and measure the distance d from the central wire out to where the mass is hung on the first crossarm..Calculate τ from the expression:

$$\tau = 980 \frac{mdl}{\theta} \cos \theta.$$

You can also measure I for your wave machine. Determine the mass M in grams and length L in centimeters of one of the crossarms. Also measure the distance D between adjacent crossarms. Then calculate I as follows:

$$I = \frac{1}{12} \frac{ML^2}{D}.$$

Now, put these measured and calculated values of τ and I into equation (3-1), and find out how fast a wave should travel on your machine. Next measure the speed of a wave directly with a watch and meter stick. Do the two speeds agree within your ability to measure the various quantities involved?

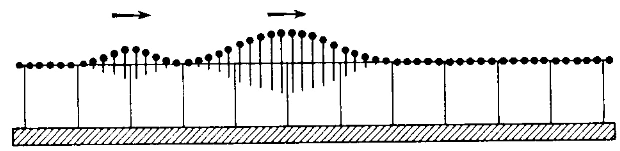

Figure 3-5. These waves stay the same distance apart as they travel along.

Constancy of Wave Speed

There are a number of factors implicit in the speed expression worth pointing out. The first of these is that the equation says nothing about the amplitude or wave length of the wave. The implication here is that waves of all sizes and shapes travel along the machine at identical speeds. Can this really be true? Try it for yourself and see. With your watch find out how long it takes little waves, big waves, long waves, and short waves to travel the length of your machine. You'll find that these times are, in fact, all the same!

Another way you can convince yourself that waves of all sizes and shapes travel with the same speed on your machine is to launch two successive waves; a little one followed immediately by a big one (see Figure 3-5). Careful observation will show that there is no tendency for one to overtake or fall farther behind the other. As they travel along they will remain exactly the same distance apart all the way. This constancy of speed means that, even though a wave dies out because of frictional dissipation as it travels, it does not speed up or slow down.

This constancy of wave speed, regardless of wave amplitude and length, is also true for waves on other mechanical systems, for sound waves, for electrical waves, and for electromagnetic waves.* How fortunate it is, too, that such independence exists. When you listen to a band playing on a football field between halves, the sound waves of various sizes and shapes corresponding to the various loudnesses and pitches of the music all travel with the same speed. This constancy enables them to reach your ears with the same relative time relationships as when they left the instruments. Can you imagine what the music would sound like if loud sounds traveled faster than soft sounds, or if bass notes traveled faster than higher pitched notes? The music would become a very uninspiring conglomeration of noise. Imagine, also, how frustrating it would be to carry on a telephone conversation if the electrical waves of different amplitudes and wave lengths which correspond to the various components of speech traveled with different speeds along the telephone wires!

In passing, I invite you to take another look at equation (3-1). On the right-hand side, we have the square root of the ratio of a quantity relating to the torsional springiness of the wave medium and another quantity relating to its rotational inertia. Similar equations can be derived for the speeds of waves on various other wave media. For example, the speed of a sound wave in a gas is:

$$s = \sqrt{\frac{1/k}{\rho}},$$

* The advanced reader is requested not to raise the quibble at this point about what happens in frequency ranges where dispersion occurs in various wave systems.

where $1/k$ is the reciprocal of the adiabatic compressibility of the gas (a measure of its longitudinal springiness), and ρ is its density (a measure of its inertia).

An electrical wave travels along a transmission line with a speed given by:

$$s = \sqrt{\frac{1/C}{L}},$$

where $1/C$ is the reciprocal of the capacitance between the wires per unit length of line (a measure of its electrical springiness), and L is the inductance along the wires per unit length (a measure of electrical inertia).

We can even apply our basic formula to clotheslines. A transverse wave travels along a stretched clothesline with a speed:

$$s = \sqrt{\frac{T}{\mu}},$$

where T is the tension in the line (a measure of its transverse springiness), and μ is its mass per unit length (a measure of its transverse inertia).

These similarities, which are summarized in Table III-1, should convince you that there is in Nature a universality, consistency, and basic simplicity which pervade all these fields of physics.

A Special Note About Water Surface Waves

In talking about the constancy of speed with which waves travel in their respective media independent of amplitude and wave length, some of you will note that I haven't even mentioned water surface waves. Can the omission have been deliberate? Indeed, it was. Water surface waves present an exception to the generalizations just covered. In their case, speed *does* depend on wave length.

TABLE III-1

Torsion machine	$s = \sqrt{\frac{\tau}{I}}$
Sound in gas	$s = \sqrt{\frac{1/K}{\rho}}$
Electric line:	$s = \sqrt{\frac{1/C}{L}}$
Stretched string:	$s = \sqrt{\frac{T}{\mu}}$

Though the symbols may vary from one expression to another, all of the above equations involve the ratio of springiness to inertia.

When you dip your finger gently into the surface of a quiet pool, the little ripples spread out with a speed of a few inches per second. When you throw a large stone into the same pool, however, the resulting waves have a longer wave length and travel much faster. Ocean rollers with wave lengths of 100 yards from crest to crest go barreling along with speeds around 15 miles per hour in deep water.

Having been persuaded in the last section that Nature is consistent, you are perfectly justified now in asking "What's so special about water waves that makes their behavior differ from that of other waves?" The answer is that the surface of a liquid is not an elastic medium in the same sense that a steel wire or a compressible gas is. There are at least two major points of difference between the behavior of a liquid surface and that of a really elastic medium.

First: When you push downward on a portion of the surface of a liquid, the liquid, being incompressible, yields by flowing sideways to escape. The displacement of the particles of the medium is not in the direction of the applied stress.

Second: Consider a unit mass of water in the crest of a wave. The downward force of gravity on this unit mass is the same regardless of whether the wave is of small or large amplitude, or of short or long wave length. In other words, the restoring force is not a linear function of the displacement of the medium as it is for media that we describe as elastic. If you twist a wire, compress a gas, or charge a capacitor; the back force opposing the applied stress is always proportional to the displacement. Not so for liquid surfaces. No wonder waves on liquid surfaces don't obey the laws on wave propagation in elastic media.

We see, therefore, that the remarks made in the previous section about wave speed being independent of amplitude and wave length must be made somewhat more restrictive. We must say, "In any homogenous, *elastic* medium, the speed of a wave is independent of its amplitude and length." If the medium isn't elastic, the statement simply doesn't apply. Note that I have sneaked in the word "homogenous" into the above generalization. This word is necessary to make the generalization still more rigorous for reasons which won't become apparent for several more chapters. For the moment bear with me and let the word stand. I promise to justify it later.

Crisscrossing of Waves

Waves going in different directions on the same medium pass right through each other. To demonstrate this generality, start two single waves simultaneously, traveling in opposite directions from each end of your wave machine. Watch what happens when they meet in the middle of the machine. They pass right through each other, don't they? Wait a minute, now. Do they pass through each other, or merely collide and rebound? Which wave is which?

You can resolve this uncertainty by starting two more waves toward each other from opposite ends of the machine. But this time make them of different amplitudes. In this way you can keep track of each wave separately. Now, there's no guesswork about what happens, is there? The waves do *not* collide and rebound; they pass through each other (or climb over each other) and continue on their merry ways as if they had never met.

Examples from your own experience of waves passing through each other are numerous. In a roomful of people talking to each other sound waves are criss-crossing in all directions. Electric waves on transmission lines can travel in both directions, quite independent of each other. In the air around your radio antenna radio waves are traveling in many directions, unaffected in their behavior by their continual encounters with other radio waves. Two pebbles thrown simultaneously into a pool a short distance apart generate circular patterns of ripples which spread out and cross through each other but do not affect each other in any way.

CHAPTER FOUR

Reflection

Reflection is one of the most generally familiar aspects of wave behavior. Optical reflection in mirrors, sound reflection from buildings or mountains (echoes), and the water-wave reflections from rigid bulkheads which cause surface chop are all part of common experience. Less familiar, perhaps, but equally common, are reflections of electric waves on lines and reflections of certain radio waves from the ionosphere and even from artificial satellites such as the high-flying *Echo*.

Now, like Socrates, let's define our terms, or at least one of them. *Reflection is the turning back of a wave upon itself when it encounters an abrupt change in the nature of the medium in which it is traveling.*

Reflection may be partial or total, depending on the severity of the change. And it may or may not be accompanied by some absorption as well.

One of the first things you probably noticed with your wave machine is that a wave, having traveled the length of the machine, reverses its direction when it arrives at the end and then travels back to its starting point. There it is reflected again, repeating a few more excursions back and forth along the machine before dying out completely.

At each reflection the wave turns around bodily, preserving both its amplitude and shape. Since the discontinuity of the machine is extreme at the dead end, the reflection is total. There is no mechanism for extracting energy from the wave in this particular act of reflection. All the energy incident with the wave is therefore reflected back with it.

Now, instead of leaving the reflecting end of the machine free as before, clamp the last crossarm so it can't be displaced when a wave comes along. Launch a wave and watch what happens when it encounters the clamped-end crossarm. Since there is nothing about a rigid clamp which can absorb any energy, we expect the reflection to be total. And it is. But watch carefully. See how it is turned upside down in the process! Free end — right side up reflection; clamped end — upside-down reflection.

The same two possibilities for total reflection occur on other wave systems as well. In every case, the kind of reflection you get depends on whether the reflecting end of the medium is free or clamped.

Reflection on Clotheslines

For example, consider the case of waves on a stretched clothesline. Take a 50-foot length of flexible clothesline and tie one end of it to a 50-foot length of light twine. Don't knot the clothesline itself. Instead, knot the twine securely around the end of the clothesline. Tie the other end of the twine to a tree, waist high. Pick up the free end of the clothesline with your hand and pull on it with enough horizontal force to lift everything clear of the ground. Launch a wave on the clothesline by giving your wrist a quick flip to your right. The clothesline now becomes a wave medium, and the other end, where the twine is tied to it, is a reflection discontinuity. Here the end of the clothesline is essentially free as far as transverse wave displacement is concerned. If the twine is light enough, almost

total reflection will take place from the junction, and the reflected wave will travel back up the clothesline with its direction of displacement also to your right.

Now untie the twine from the clothesline, and tie the clothesline directly to the tree. Launch another wave to the right on the clothesline. This time, because the other end of the line is effectively clamped, the totally reflected wave will come back toward you with its direction of displacement to your left!

Sound Reflection

Similar behavior is exhibited by sound waves traveling inside a tube. If the tube is left open to the air at the far end, it may be considered to be free-ended* as far as longitudinal air waves are concerned. A pulse of condensation traveling along such an open-ended tube is reflected as a pulse of rarefaction traveling in the opposite direction. The displacement of air layers in the tube is in the same direction for a reflected pulse of rarefaction traveling west as for the original pulse of condensation traveling east.

If the end of the tube is closed by a rigid baffle which prevents the longitudinal displacement of the layers of air immediately adjacent to it, the air column in the tube is effectively clamped at the far end, and the reflection taking place at that end should be of the in-

* The reflection taking place from the open end of an acoustic tube is only partial, but this circumstance does not alter the argument about which way around the direction of displacement of the medium should be. To make a totally reflecting open-ended tube, you would have to cover the end of the tube with an airtight massless elastic membrane and provide a vacuum chamber on the outside of the membrane, as shown in one of the sketches of Figure 4-1.

verted variety. As is well known, a pulse of condensation traveling down a closed tube is totally reflected as a pulse of condensation traveling in the opposite direction. In this case, the longitudinal displacement of air in the tube is in the opposite direction from that of the original pulse before reflection.

Reflection on Electric Transmission Lines

Analogous behavior is found for total reflection at the end of an electric transmission line. If such a line is short-circuited at the far end, the reflected wave comes back without inversion. If the other end is left open-circuited, the returning reflected wave is inverted. A short-circuit termination of a transmission line is thus analogous to the free end of a mechanical wave medium. It is a place where unrestrained displacement of the medium (in this case, the electrons in the conductor) may occur. If the termination of a transmission line is open-circuited, it acts like the clamped end of a mechanical wave system. Electrons cannot flow through the open circuit, and so are effectively clamped.

Water Reflection

In the case of total reflection of water surface waves from the vertical face of a rigid bulkhead, the reflection occurs without inversion of the vertical component of the wave motion. The bulkhead presents no restraint to vertical motion of the water surface and may, therefore, be regarded as a free-ended termination of the wave medium. Thus, reflection without inversion is an expected result.

Optical Reflection

When an electromagnetic wave in a beam of light is reflected by the smooth surface of a metal mirror, it is inverted by the reflection. In a continuous train of optical waves, the reflected waves, as they leave the surface of the mirror, are 180 degrees out of phase with the incoming waves which produce them. To a beam of electromagnetic waves, the mirror behaves like a clamp at the end of the portion of space occupied by the incident beam.

In the case of optical reflection from practical metal mirrors, some absorption always occurs, and the reflection is less than the 100 per cent ideal. For mirrors of silver, rhodium, aluminum, etc, the absorption loss is small and can be neglected. On the other hand, for metals such as iron, tin, nickel, and lead, the absorption is considerable. For other metals the absorption may differ in different wave-length regions of the visible spectrum. Thus, copper appears red; gold, yellow; and so on.

If you remember that a metallic short-circuit across the end of a transmission line behaves like a free-ended termination, it may seem terribly confusing now to be told that a metallic mirror across the end of a beam of light behaves like a clamped-end termination! Why the difference? In replying, let me say that I'm aware of your possible confusion but am unable to resolve it without going into a long and involved explanation. For the moment, let's concentrate only on the empirical and behavioral similarities of the reflection process in various wave systems.

The similarities of reflection behavior which we have developed so far can now be summarized in such a tabular fashion as to bring out the analogous features of different reflecting terminations of different media. Figure 4-1 does this. Here we have three columns. The first lists various types of wave in various media. The second lists and illustrates for these various wave systems the appropriate terminations to produce right side up total reflection. The third lists and illustrates terminations which produce upside-down total reflection.

In these diagrams the solid lines and arrows indicate

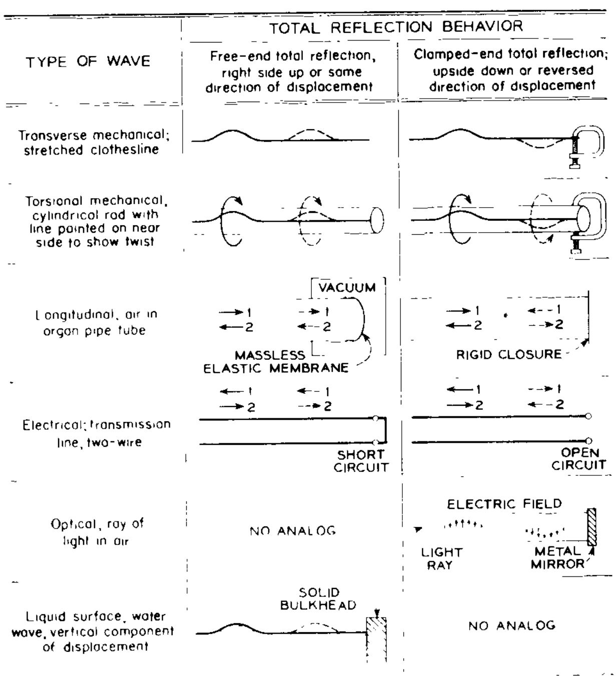

Figure 4-1. Totally reflecting terminations produce similar effects in all these wave systems.

the displacement of the particles of the medium for a pulse traveling from left to right before reflection. The dashed lines and arrows refer to the displacement of the particles of the medium for the pulse after reflection.

In the diagrams for sound waves in pipes, the numbers 1 and 2 beside the solid arrows refer to the directions of displacement and subsequent restoration of molecules in a lamina of air as a compressional pulse travels from left to right in the tube before reflection. Similar numbers beside the dashed arrows indicate the directions of displacement and subsequent restoration in the reflected pulse coming back.

In the diagrams for electric waves on a transmission line, the numbers beside the solid arrows indicate the directions of displacement and restoration of the electrons in the upper wire as a positive electric pulse travels along the wire from left to right. Numbers beside the dashed arrows indicate the direction of displacement and restoration as the reflected pulse comes back.

In all diagrams the reflecting termination is at the right.

As you scan Figure 4-1, you may be puzzled by what appears to be a hopeless paradox: You'll notice that a *closed*-ended acoustic tube gives reflection behavior similar to that of an *open*-circuited electric transmission line. You may even be thinking, "How inconsistent physicists are with words and meanings!" Perhaps; but the expressions "open tube," "closed tube," "open circuit," and "short circuit" weren't really invented to confuse you. They grew up at different times in different physical disciplines, and their originators probably weren't even aware of the terminology being evolved in parallel fields. It may help your spirit of tolerance if you keep in mind that scientists themselves have their human inconsistencies.

Another Special Note About Water Surface Waves

If you fasten your attention on the motion of a particle in the surface of a deep body of water as a continuous train of sine waves passes by, you will find that the particle executes a complete circular path during each wave period. It will surge forward, upward, backward and downward as each wave passes beneath it. This circle is executed counterclockwise for a wave traveling to the right (Figure 4-2).

Now, a circle can be obtained by composition of two simple harmonic motions — one up and down, and the other back and forth, each being a quarter-period out of phase with the other. You may, therefore, describe a water surface wave as a combination of transverse and longitudinal modes of vibration. When such a

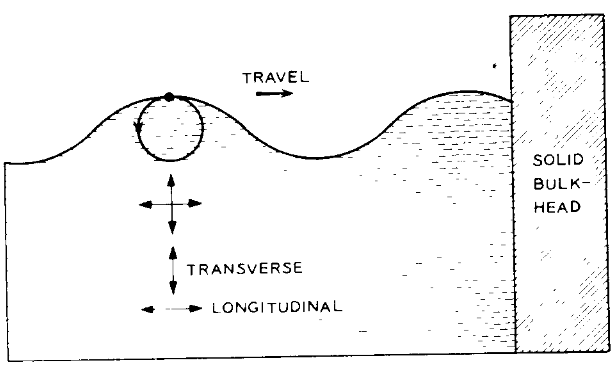

Figure 4-2. A particle in the surface of the water, executes a circular counter-clockwise motion, which can be resolved as the resultant of a vertical tranverse and a horizontal longitudinal simple harmonic motion, a quarter-period out of phase with each other.

wave encounters a rigid vertical bulkhead, there is no restraint on the vertical transverse component of the wave motion. This component is reflected without change of phase, since the bulkhead acts like a free-ended termination to the wave medium.

The horizontal longitudinal component of the motion, however, sees a rigid obstacle to horizontal displacement. To this component the bulkhead acts like a clamped-ended termination, and the horizontal component of the wave motion is reflected with a 180 degree change of phase. In the reflected wave, then, we have the vertical and the horizontal components of the particle motion once again a quarter-period out of phase with each other. But this time the phase of the horizontal component is 90 degrees behind that of the vertical component instead of 90 degrees ahead of it. The circular motion of the particle in the water surface is, therefore, clockwise, rather than counterclockwise. This reversal of direction of the particle's circular motion is consistent with the reversal of the direction of travel of the wave upon reflection.

CHAPTER FIVE

Superposition, Standing Waves, and Resonance

Part of a responsible scientist's job is to be ever alert for common denominators in the behavior of things in Nature. If he finds similar features or behavior patterns in a considerable body of varied natural phenomena, he asks, "Are the similarities due to some basic law which underlies all these situations?" If, after further experimentation and observation, he is convinced that there is an underlying law, he formulates a carefully worded statement called a hypothesis, which he publishes for other scientists to read, think about, and apply to their own particular situations. If, over a period of many years, no exceptions to the hypothesis turn up, and if the application of the idea embodied in the hypothesis makes it possible to predict things which are subsequently tested and found to be true, then the hypothesis is accepted by scientists as a law or principle. It has stood the test of time.

Superposition

One such underlying law is the principle of superposition. As we shall see, it relates a number of different wave phenomena to a single unifying idea.

The principle of superposition states that when two waves meet each other on the same medium, the instantaneous displacement of the medium is given by

the algebraic sum of the instantaneous displacements of the individual waves. Upon this principle depend a number of phenomena which we shall presently discuss. These phenomena include interference, standing waves, and resonance.

The validity of the principle can easily be demonstrated with your wave machine. Start two identical symmetrically-shaped single waves traveling toward each other from the opposite ends of the machine. Notice that when they meet in the middle the amplitude of the single crest resulting at the instant of exact superposition is clearly larger than the individual amplitude of either wave alone. How can you prove that it is exactly twice? One way is simply to observe the progression of the waves against a scale consisting of a number of parallel equally-spaced horizontal lines. Make a quick visual estimate of the amplitudes of the individual waves before they meet and a similar estimate of the resulting amplitude at the instant of exact coincidence. Is the latter the algebraic sum of the former? If you have measured correctly, it will be.

A more elegant method of demonstration would be to take a photographic time-exposure of the wave machine. The exposure should begin before the two

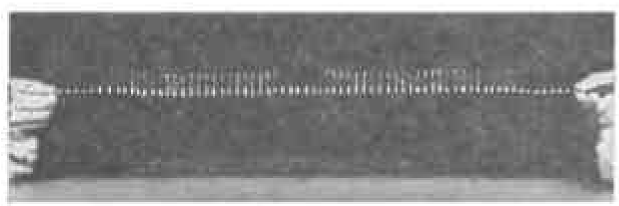

Figure 5-1. This time exposure photograph shows the amplitudes of two separate positive waves and, in the middle, of their superposition resultant.

waves meet and continue until after they have crossed over each other and continued on their separate ways. Such a time-exposure photograph is shown in Figure 5-1. In it you can easily identify the individual wave amplitudes and the resulting superposition amplitude. The latter is indeed equal to the sum of the former.

If you have difficulty in making two waves reasonably identical in amplitude, shape, and symmetry on your wave machine, try the following: At the middle of the machine, give the end of one of the crosspieces a quick up-and-down displacement. Identical waves will start outward in opposite directions from this point. They will then be reflected from the two ends of the machine and return to meet in the middle. With a little practice in starting waves this way, you'll learn how to make them symmetrical in form, with their trailing faces shaped exactly like their leading faces.

Superposition with Waves of Opposite Displacement

The principle of superposition is equally valid for the superposition of waves having opposite directions of displacement. Again, start single waves simultaneously from opposite ends of the machine, making them as symmetrical in shape and equal in amplitude as you can. But, this time, make one with its displacement upward and the other with its displacement downward. When the waves meet in the middle of the machine, there will be an instant — the instant of exact coincidence — when the displacement of the machine is zero. This condition is fleeting, to be sure. But a snapshot of the machine taken at this instant would show all the crossarms in their normal undisplaced positions. From such a snapshot, you couldn't tell whether you are looking at the resultant cancellation of identical positive and negative waves, or at a machine

Figure 5-2. In this time exposure a positive wave traveling left to right and a negative wave traveling right to left have canceled each other in the middle of the machine.

with no waves on it at all! Figure 5-2 is a time-exposure of the wave machine showing a positive and a negative wave approaching each other, crossing through and cancelling each other, and resuming their original amplitudes and directions. The cancellation at the exact instant of superposition can be clearly seen as the gap in the wave smears in the middle of the machine.

Figure 5-3 shows a sequence of motion pictures of a wave machine with positive and negative waves approaching each other. In this sequence the fourth picture has caught the action at the exact instant of cancellation. As you see, the resulting displacement has vanished. However, after the waves pass through each other they resume their original shapes and continue on their original independent ways unchanged by the experience of meeting one another.

You may already have figured out for yourself that a more sophisticated way of getting two identical waves of opposite sign to travel toward each other is to clamp one end of the machine, leave the other end free, and start two identical waves traveling towards the ends by a single up-and-down displacement of the middle crosspiece. The wave arriving at the clamped end will be reflected upside down, while the wave reflecting from the free end will return right side up.

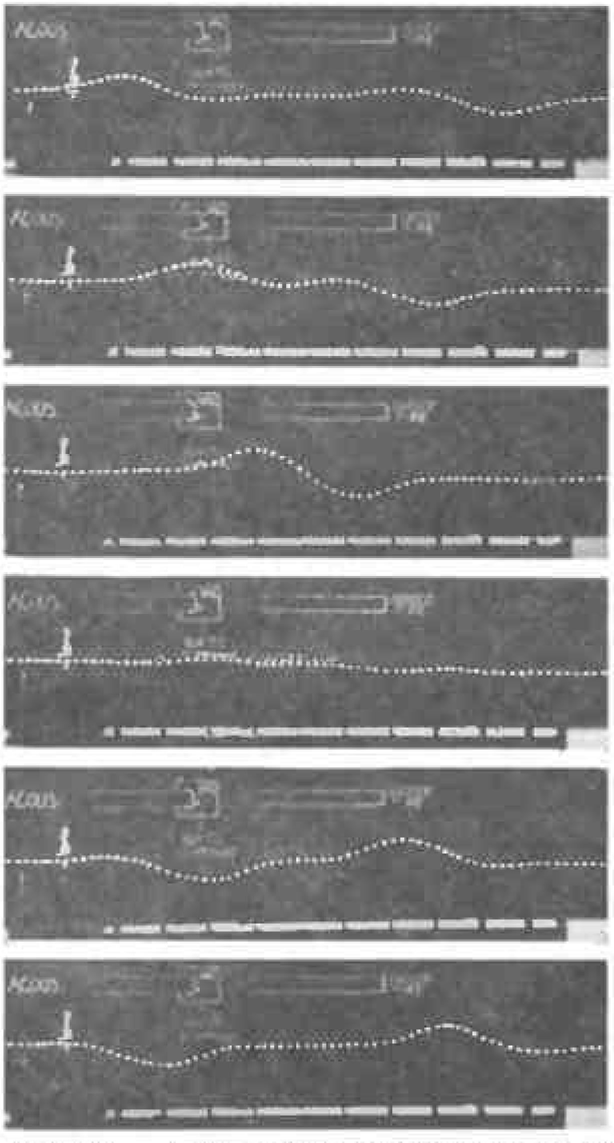

Figure 5-3. In this sequence of motion pictures positive and negative waves have canceled each other at the exact instant of superposition.

The positive and negative waves will then meet in the middle, where, if you were successful in making them symmetrical, the cancellation will be complete at the instant of coincidence.

Interference and Standing Waves

Now that we've considered the superposition behavior of single waves or pulses, we are ready to talk about the interference of trains of continuous waves moving in opposite directions on the same medium. But, before we do any demonstrating, let's try to use the knowledge we've already acquired to predict what to expect.

Imagine that two trains of continuous waves having equal amplitudes and wave lengths are traveling through each other in opposite directions. Specifically, imagine them as mechanical waves on your wave machine. Let the positions of these two trains at some particular instant be those of the dashed lines A and B in Figure 5-4(a). You won't see either of these two wave trains on the machine. What you will see at this instant is their superposition resultant, which is given in the same figure by the solid line obtained by algebraically adding the displacements of the individual wave trains from point to point along the medium. This resultant displacement is what a snapshot would show if you photographed the wave machine at this instant.

Now suppose we were to take another snapshot exactly one-twelfth of a wave period later. During the preceding interval each of the two wave trains A and B has advanced one-twelfth of a wave length farther along in its particular direction of travel. The new positions of A and B are shown by the dash lines in

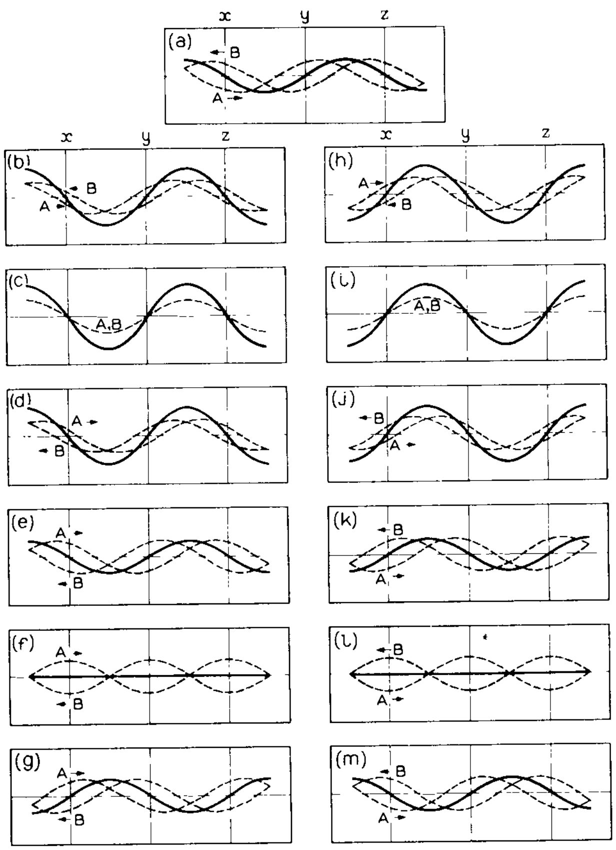

Figure 5-4. As wave trains A and B assume the positions shown at successive one-twelfth-period intervals, the medium itself assumes the standing-wave displacements shown by the successive positions of the solid line.

Figure 5-4(b). Their instantaneous resultant is indicated by the solid line in the same figure. This resultant has a larger amplitude than it had in the first snapshot. However, note that the resultant hasn't moved either

to the right or to the left in the one-twelfth period interval. It has merely changed in amplitude.

Passing now to Figure 5-4(c), which sketches the state of affairs still another twelfth of a period later, we see that wave trains A and B have moved into exact coincidence with each other. The resultant is now of still larger amplitude than in either of the two preceding figures; but, as before, it has moved neither to the right nor to the left.

Now carefully examine the remaining ten sketches in Figure 5-4. Each one gives the instantaneous positions of A and B and of their resultant at successive intervals of one-twelfth of a period. Note the progressive changes in the amplitude of the resultant. Observe, too, that in sketch (f), A and B have moved to positions of opposition so that they cancel each other everywhere along the medium. In this case, the resultant coincides with the undisturbed aspect of the medium. In sketch (i), the resultant has attained a maximum displacement in the opposite direction everywhere from the displacement it exhibited in sketch (c), half a period before.

In sketch (l), the individual wave trains A and B are once more in exact opposition, and the resulting displacement of the medium is again zero everywhere. Finally, in sketch (m), the action has gone through one complete period and everything is exactly as it was in sketch (a). The entire sequence we have just described is now ready to repeat itself again and again.

To sum up: What we predict is a resultant displacement which continually changes its amplitude, going from a maximum in one direction through zero, to a maximum in the other direction and then back again, cycle after cycle. The resultant, however, does not move in either direction along the medium. Moreover, it exhibits a succession of dead spots spaced half a wave length apart where the superposition of the constitutent wave trains is such as to produce complete cancellation all the time. These dead spots, or *nodes*, are indicated by the loci, x, y, and z in the thirteen sketches of Figure 5-4. Incidentally, the region of the wave pattern enclosed between any two adjacent nodes is called a *loop*.

Now let's go to the wave machine and see if our predictions will be borne out when put to test.

Two identical wave trains going in opposite directions can be produced by simply generating continuous waves at one end of the machine with the motor and crank attachment. When these waves are totally reflected from the opposite end the going and returning waves pass through each other on the machine.

The wave pattern first appearing after reflection begins to take place may be unrecognizable for a time.* Gradually, however, things will settle down, and a steady-state wave pattern will emerge which exhibits all the features we have predicted.

* Whenever you suddenly start doing something to a wave system which you weren't doing before, or whenever you suddenly stop doing something you were doing before, the system is thrown into a temporary state of disorganization called a *transient*. This transient dies out after a time — perhaps a quarter of a minute or so in the case of the wave machine. When the transient has disappeared, the true steady state response of the system to whatever type of wave generation you provide emerges and continues as long as excitation is continued. Each time you change a condition of excitation, such as amplitude, frequency, phase, or point of attachment to the wave source, you start a new transient on the machine. The standing wave patterns we are talking about are steady-state patterns which should always be observed after the disappearance of whatever transients are occasioned by changing any of the conditions of the machine.

Note the peculiar appearance of this pattern. It consists of a number of segments separated by nodes which simply bob up and down without going anywhere along the machine. In contrast with running waves, which travel along the machine, these are *standing waves* formed by the interference of the two wave trains going in opposite directions. Just as the nodes are produced by the destructive interference of the waves in the two trains, the maximum amplitude of the loops midway between each pair of nodes is produced by the constructive interference of the waves in the two wave trains. At these midway points the amplitude of the loops is twice the amplitude of the running waves in the individual wave trains.

Figure 5-5 is a time-exposure photograph of such a standing-wave pattern on my wave machine. While the nodes appear at regularly-spaced intervals half a wave length apart, the position of the nodal pattern with respect to the machine's ends depends on how you chose to produce the reflection at the terminating end. If you employed a free end as the reflector, the node nearest this end appeared a quarter of a wave length away toward the generator. But if you used a clamp for the reflector, as I did for Figure 5-5, the clamped end became itself a node, since the clamp enforced a

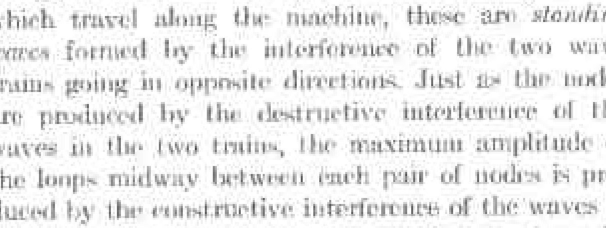

Figure 5-5. This time exposure of a standing wave suggests the segments of a vibrating string.

condition of zero displacement. The next nearest node, therefore, appeared half a wave length away from the end.

Note that by using the clamp as a reflector you can chase the entire pattern of nodes up the machine toward the generator by moving the position of the clamp in that direction. Each time the clamp is moved, wait for the transient to die out before making your observation. No matter where you place the clamp, the nearest node will always be half a wave length away, and all other nodes will position themselves accordingly at successive half-wave length intervals. Note, also, that the point of attachment of the generator doesn't have any effect on the positions of the nodes. Obviously, then, the location of the nodes depends only on the nature and positioning of the reflector. The distance between reflector and generator merely determines how many nodes there will be.

Resonance

While you were experimenting with different placements of the generator and reflector, you must have noticed that the amplitude of oscillation of the loops of the standing waves varied from case to case. Let's examine this variation systematically.

Attach the generator to the first crossarm and leave it there throughout the following procedure. Now attach the clamp to the last crossarm, and note the standing-wave amplitude which persists after the transient has died out. Then move the clamp just far enough up the machine to drive the node nearest the generator to within three or four crossarm spacings' distance of the generator crossarm. Again note the standing-wave amplitude. Now move the clamp, a

single crossarm at a time, closer to the generator, noting the steady-state standing-wave amplitude after each change. As the node is driven closer to the generator crossarm, the standing-wave amplitude increases. In fact, you'll have to be careful not to allow the amplitude to become so large the machine comes in danger of jumping out of its bearings, or of twisting the central wire beyond its elastic limit.

At this point, the system is in a condition of *resonance*. The abnormally large amplitude is an extreme example of superposition. The length of the machine is now equal to (or very nearly equal to) a whole number of half-wave lengths. This correspondence means that the travel time of a wave from the generator down to the reflector and back again is a whole number of wave periods. Thus, a wave, starting at the generator and reflected back and forth along the machine between generator and reflector, always arrives back at the generator in just the right phase to superpose itself upon a new wave being sent out by the generator at that instant. The new waves and the previously emitted, multiply reflected waves thus superpose constructively, crest on crest, as they travel back and forth on the machine.

Because of this constructive superposition, the amplitude builds up until the rate of frictional energy loss, which becomes larger as the amplitude increases, is equal to the rate of new energy input from the generator. A transient condition exists while this buildup is taking place, but when the build-up interval is finished, the steady-state amplitude of the resulting standing wave is several times larger than the amplitude of any of the individual waves which produce it.

The process of adjusting the length of the wave machine to a whole number of standing-wave loops in order to produce resonance is called *tuning*. Tuning the wave machine to resonance could have been accomplished just as well by leaving the machine at its original full length and varying the frequency of the generator until the half wave length of the waves fitted a whole number of times into the length of the machine. Resonance occurs on any standing-wave system when the length of the medium and the wave length of the waves traveling upon it are related by the expression:

$$L = \frac{n\lambda}{2},$$

where L is the length of the medium between generator and reflector, λ is the wave length, and n is any whole number, 1, 2, 3, $\cdots$.

Another expression:

$$L = \frac{ns}{2f}$$

says the same thing. Here, s is the wave speed in the particular medium and f is the frequency of the waves.

Resonance can be produced on any kind of wave system by adjusting the length of the system or the frequency of wave generation so as to fulfill the above conditions. Many people like to sing in the bathtub because of the resonant quality produced in the notes of certain pitches. The air in a bathroom is a three-dimensional wave medium, with its dimensions set by the spacing of walls, ceiling, and floor. Of course, you can't move the walls around to produce acoustic resonance. But you *can* sing up and down the scale, varying your frequency and wave length so as to match

the particular frequencies and wave lengths that produce resonance between the side walls, between the ends of the room, and between the ceiling and floor.

Bathrooms are particularly well suited for the resonant enhancement of sound because of their "live" qualities. Of all rooms in a typical house, the bathroom is least likely to contain sound-deadening materials — carpets, draperies, upholstered furniture, and the like. Consequently, sound waves are reflected many times before dying out. At each reflection, superposition may occur, thereby increasing the amplitude of the sound.

Resonance and Water Surfaces

The behavior of water in a rectangular basin affords another interesting example of resonance. When you tip the basin slightly, you generate a wave of water which travels to the other end and is reflected. If you restore the basin to a level position at the instant the wave is reflected and then tip the basin again when the reflected wave returns to your end (repeating these operations each cycle), you will gradually build up a large resonant amplitude of oscillation of the water even though the amount of tipping may be very slight. What you are doing, of course, is tuning the frequency with which you tip and untip the basin to the frequency at which the wave sloshes back and forth. There will be a node of level amplitude across the middle of the basin, while the ends, since they prevent the horizontal flow of liquid, become nodes of current.

If you increase the size of the basin until you have a body of water about 75 miles long and 25 miles wide, and if you simply raise and lower the level of water at one end instead of tipping the whole basin back and forth, you will have a body of water which behaves like the Bay of Fundy in New Brunswick. This bay is of such a size as to be fairly well tuned to the period of its tides. Consequently it becomes a resonant system. The 40-foot amplitude of the water level at the head of the bay is the resultant of the resonant superposition of the waves from several previous tidal fluctuations. The shape of the bay, too, suggests that some step-up transformer action may be present as well.

Resonance on Electrical Transmission Lines

Standing waves and resonance on electrical ac transmission lines also offer some interesting and important considerations. Resonance on a power transmission line can have serious consequences. Suppose a hydroelectric power generator station is located far from the city to which it furnishes power by means of an overland transmission line. During a storm a tree falls across the line and open-circuits it by breaking both wires. If the break occurs at a place one-quarter of a wave length (or three-quarters, or five-quarters, etc.) away from the generator station, the line becomes resonant. In a few cycles the amplitude of the voltage at the open circuit will build up high enough to flash over at the insulators. Back at the generator station, enormous currents will begin flowing in the generator. If nobody pulls the switch, the generator will soon burn up its insulation and begin throwing out melted copper!

In a well-designed power system, automatic circuit breakers come into play in situations like this before any damage can be done. But, even so, you can appreciate that power-line systems should be designed by people who know their way around in wave physics.

CHAPTER SIX

The Impedance Concept
Impedance Matching and Mismatching
Partial Reflection

Whenever you have a wave medium on, in, or through which waves are being propagated, this propagation can be described in terms of two quantities — *cause* and *effect*. For example: To launch a train of waves on your wave machine, you grab the end of the first crossarm and repeatedly move it up and down. This oscillating vertical force produces an oscillating torque on the central wire. The oscillating torque, in turn, is the *cause* of the waves. Successive portions of the wave machine respond by executing an oscillating angular displacement and assuming a corresponding oscillating angular velocity. This oscillating angular velocity, imparted in turn to each crossarm along the length of the machine, is the *effect*.

In wave theory the ratio of cause to effect is of particular significance. The term *impedance* is used to designate this ratio:

$$\text{Impedance} = \frac{\text{Cause}}{\text{Effect}}.$$

If the crossarms of your wave machine are very long or very heavy, or if the central wire is very stiff, a relatively large oscillating torque will be required at the first crossarm to produce even a small oscillating angular velocity of the crossarm structure. Such a wave machine would have a large impedance. On the other hand, if the crossarms are short or light, or if the central wire is very thin and flexible, a relatively small oscillating torque applied at the first crossarm will produce a relatively large oscillating angular velocity of the crossarm system. Such a wave machine will have a relatively small impedance. The impedance of a wave medium, then, can be used as a characterizing parameter to denote the ease or difficulty with which waves can be launched upon it.

The concept of the impedance of a wave medium is perfectly general. It can be applied to media of all kinds. In fact, the term originated in the field of electrical engineering where it is used to denote the ratio of the ac voltage applied at the input end of a circuit or transmission line to the ac current which flows in that circuit or line. It is probably in this reference that the term is most familiar to you — that is, if you have ever encountered it before at all.

By applying the concept of impedance as the ratio of a wave-generating cause to a wave effect, we can define the impedances of wave media of all kinds. The impedance of a stretched clothesline on which a train of waves is being launched would be the ratio of the oscillating transverse force applied at the input end to the oscillating transverse velocity imparted to the first portion of the clothesline. The acoustic impedance of a column of air in a pipe is the ratio of the alternating compressional and rarefactional forces acting on a lamina of air in the pipe to the backward and forward oscillating velocity imparted to this lamina. The impedance of empty space to the propagation of electromagnetic waves is the ratio of the oscillating electric field to the oscillating displacement current* density in the empty space.

The term "impedance" is very similar in meaning to the term "resistance," which is used to denote the ratio of cause to effect in cases where motion is unidirectional rather than oscillating. In the case of an automobile whose wheels are capable of producing a constant forward driving force, the automobile speeds up until, at some particular speed, air resistance and other frictional effects exert a counter force equal to the driving force. The speed thereafter remains constant, and we may properly speak of the resistance of the system as the ratio of the driving force to the terminal velocity.

The engines of a ship exert a certain torque on the propeller shaft. Consequently, the propeller rotates at a certain angular velocity. We may speak of the resistance of the propeller as being the ratio of the torque to the resulting angular velocity.

The resistance of an electrical resistor is the ratio of the dc voltage across its terminals to the dc current flowing through it as a result. For most intents and purposes, the numerical value of the dc resistance of a resistor is the same as the value of its ac impedance.

Impedance Matching and Mismatching

One of the more important concerns of modern electric power technology is the efficient transmission of power from a generator to the place where it will be used. This transmission is done by means of electrical

* A displacement current is a current which can flow even in regions where no moving charges exist to carry the current. The theory of displacement currents is treated in advanced courses in physics.

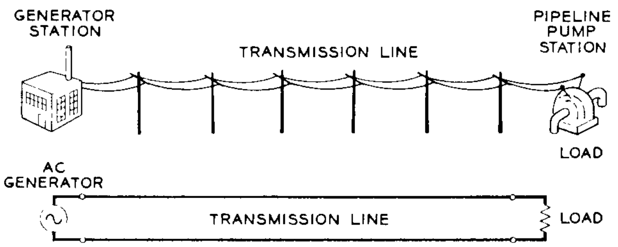

Figure 6-1. The real situation sketched above is represented by the symbolic diagram below.

waves on transmission lines. Figure 6-1 is a simple sketch of such a system.

When you wish to transfer power efficiently from a line to a load,* you don't want a lot of reflection to take place at the load end of the line. Power reflected is power lost from the load. In earlier chapters we discussed extreme cases of total reflection by open-circuited (infinite impedance) and short-circuited (zero impedance) loads at the ends of various kinds of transmission media. But what about *finite* impedance loads?

It can be shown both theoretically and experimentally that when a transmission line is delivering power to a load, the delivery is maximum when the impedance of the load is made equal to the impedance of the line. If this condition is fulfilled, all the wave energy traveling along the line is completely absorbed by the load, and there is no reflection whatsoever. The load is then said to be *matched* to the line. If the impedance of the load is different from that of the line, only part of the

* The word "load" is employed to denote the thing at the end of the line which uses the power. An electrical load can be a lamp, a motor, a home appliance, or many such things in parallel.

wave energy will be absorbed by the load, and the rest will be reflected back toward the generator. The amount of energy which is thus lost by reflection increases as the impedance mismatch between load and line becomes greater. In the case of extreme mismatches (infinite or zero impedance loads) the energy absorbed is zero and the reflection is total, as we have already seen.

How to Demonstrate Impedance Matching and Mismatching

All of these impedance matching and mismatching consequences can be demonstrated beautifully with your wave machine.

To begin, you'll need a mechanical load. This is a simple affair which can be made from a tin can, a sheet of metal, a piece of wire, and a dab of solder. Figure 6-2, shows how to assemble these parts into what amounts to a dash-pot-and-piston arrangement not unlike the shock absorbers in an automobile.

The piston must fit very loosely in the can. There should be no friction between the rim of the piston and the walls of the can. Energy absorption should be due solely to the pumping of water back and forth around the edge of the piston. The piston and wire assembly should be made as light as possible, consistent with stiffness, in order not to tip the wave machine off balance when it is clipped onto the last crossarm. If you pump the piston assembly up and down with your hand, you can feel the counter force with which it resists your effort. When clipped onto the last crossarm of your wave machine, it becomes an energy-absorbing load.

The impedance which this load presents to the wave

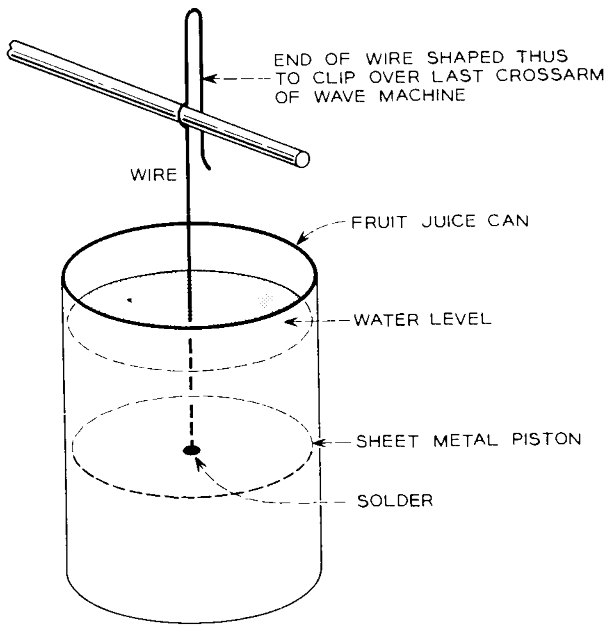

Figure 6 2. A can of water with a loose piston makes a good mechanical impedance termination for the wave machine.

machine depends on how far out it is attached on the last crossarm. Obviously, if it is clipped on very close to the central wire, it will operate on a short lever arm and have small effect. On the other hand, if it is clipped on near the end of the crossarm, the lever arm will be longer and the counter-torque with which it opposes the displacement of the crossarm will be larger. Its impedance is, therefore, higher. Unless you have made the can and piston much too small, there will be some point of attachment along the crossarm where its termination impedance will be equal to the impedance of the wave machine.

To find this point, proceed as follows. Start with the dash pot attached to the last crossarm, about an inch out from the central wire. Manually, launch a single pulse at the input end of the machine and observe approximately what fraction of its original amplitude is reflected at the dash-pot end. Make sure this reflection is right side up. Now move the attachment of the dash pot out along the last crossarm. Launch another pulse, and you'll see that the amplitude of the reflected fraction of the pulse is smaller than before. The reflection is only partial. Repeat this procedure until you locate the proper point of attachment at which all the pulse energy is absorbed by the dash pot and no reflection at all occurs. The impedance of the load is now matched to that of the machine.

The electrical counterpart of this condition is the one sought after by telephone engineers to minimize bothersome reflections and echoes on long-distance telephone circuits. Its acoustic counterpart is sought after by acoustic engineers in designing sound-absorbing coverings for the walls of rooms which must have no echoes or reverberations.

With the load in this matched position, connect the motor and eccentric drive crank to the input crossarm and send continuous waves down the line. Observe the succession of wave crests marching along and disappearing at the dash-pot end as they deliver all their energy to the dash pot. The absence of reflection means there will be no evidence of standing waves on the machine. You should see only one-way running waves.

Now detach the dash pot and re-attach it out near the end of the last crossarm. So placed, the dash pot has a higher impedance than the wave machine, as you can verify with either continuous waves or hand-

TABLE VI-1

Each of the last three columns lists terminations producing analogous reflection behavior in the wave systems listed on the left

Type of Wave System	Type of Termination: Perfectly Matched: no reflection	Mismatched: partial reflection	Totally Mismatched: total reflection
Torsion Wave Machine	Dash pot in matching position	Dash pot at some other position	Last crossarm completely free or completely clamped
Sound Beam	Wall covered with thick layers of loose glass wool	Ordinary wooden wall	Solid masonry wall
Electrical Transmission Line	Exactly matching electrical impedance load	Load of some other impedance	Zero or infinite impedance load (short- or open-circuit termination)
Light Beam in Air	Black velvet	Glass surface	Metal mirror
Water Surface	Very gradually sloping beach; waves break completely	Steeply sloping beach; waves break partially	Solid vertical sea wall

launched pulses, and partial reflection occurs again. But, this time, the reflected waves are upside-down rather than right side up. And so they should be, since the dash pot behaves more and more like a clamp the farther out on the crossarm you attach it. If it were possible to have an infinitely long crossarm with the dash pot out at the very end of it, you would effectively have a complete clamp. You would then expect total reflection of the upside-down variety.

Table VI-1 gives a listing of the various impedance terminations which one may use for loads at the ends of various wave media to obtain perfect matching, mismatching, or extreme mismatching that will result in 100 per cent reflection.

CHAPTER SEVEN

Partial Reflection at the Boundary Between Two Media

In the last chapter we saw that the amount of partial reflection of waves at the load-terminated end of a wave medium depends on the impedance mismatch between load and medium. The greater the mismatch, the greater the reflection. Now we're ready to develop the idea that partial reflection also occurs when the output end of one propagation medium is connected to the input end of a second propagation medium having a different impedance. In such a case, the input end of the second medium may be looked upon as a mismatching load at the end of the first medium.

Demonstration

To demonstrate the truth of this generalization, take the long-crossarm wave machine and connect it end to end with the short-crossarm machine. The connection

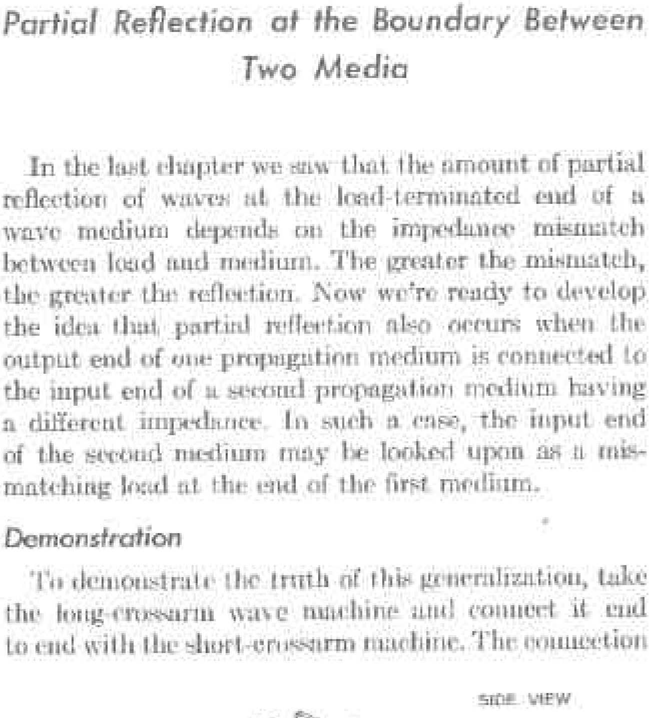

Figure 7-1. The two central wires can be connected end to end by a simple screw clamp.

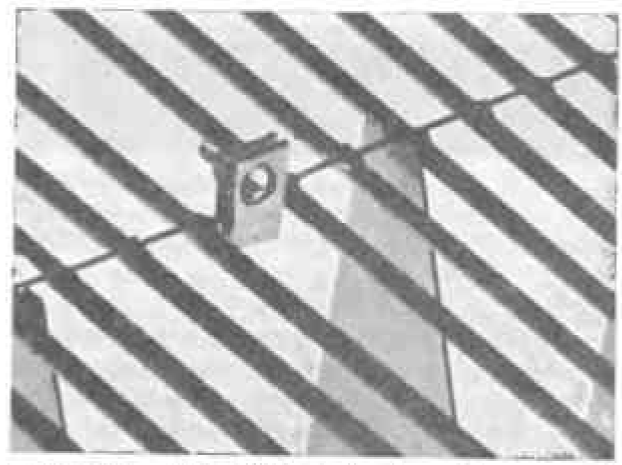

Figure 7-2. This photograph shows the connecting clamp in place.

can be made by bending up the last quarter inch of the central wires of both machines and clamping the turned-up ends with a screw clamp as shown in Figure 7-1. The screw clamp can be made with two pieces of sheet metal about half an inch square, drilled through their centers. The holes accomodate a bolt which clamps the two pieces together on the bent-up ends of the central wires of the two machines. Figure 7-2 shows a photograph of this connection on the wave machines I have built.

Now you have a single transmission medium made up of two sections having different impedances because of their different crossarm lengths. Launch a short pulse on the input end of the long-crossarm machine. Observe that the pulse is partly transmitted across the connection to the second machine and partly reflected from it back to the first machine.

As the transmitted portion of the pulse enters the second machine it speeds up to a speed appropriate to

Figure 7-3. A is the original pulse before reflection. B and C are the partially reflected and partially transmitted portions of A after it has traveled to the connecting point of the two wave machines.

the second machine and lengthens out accordingly. The reflected portion of the pulse returns right side up as befits reflection at a lower impedance mismatch. It is diminished in amplitude and has a speed appropriate to the first machine. Figure 7-3 is a double-exposure photograph depicting (A) the original pulse, and (B) and (C) the reflected and transmitted portions shortly after the original pulse has arrived at the connection point.

In your own experiments you will be able to make cleaner observations if you first connect the dash pot in a matching position at the output end of the second machine. Doing so will kill the reflections which would otherwise occur there and will thus make it easier to follow the reflection and transmission of the main pulse at the connection between the two machines.

The partial transmission and partial reflection of waves which you have just observed is the mechanical counterpart of many other physical situations with which I'm sure you are familiar. For example, when a beam of light passes from air into water, the beam is partly reflected at the surface because air and water are different media. A physicist would say they have different indices of refraction; an engineer would say they have different impedances.

We find the same phenomenon in acoustics. Sound is partly reflected after traveling down a tube when it emerges from the open end of the tube into free air. The acoustic impedance of a confined air column in a tube is different from the impedance of the free air beyond the open end.

The same phenomenon comes up in telephony, too. When long-distance telephone wires are gathered into an underwater cable, after running overland on telephone poles, the change in impedance can create undesirable reflections of telephone signals unless particular attention is paid to the impedance design of both the open wires and the cable. In general, the impedance of a free wire in air is different from that of a wire in a cable.

SWR — A Useful Ratio

Now, instead of using single pulses to investigate the partial reflection at the connection point of the two wave machines, attach the motor and eccentric crank to the input crossarm of the first machine and send out continuous waves. Before you start the motor, attach the dash pot in a matching position to the last crossarm of the second machine. Doing this will prevent unwanted reflections from hashing up the picture I want you to see. Now start the motor and watch the wave pattern which develops on the first machine. This pattern should be the superposition resultant of waves traveling down to the connecting point and waves of the same period and wave length, but of smaller amplitude, returning in the opposite direction. You should, therefore, expect some sort of standing wave on the first machine. And, indeed, if you look sharply, you will see a standing-wave pattern consisting of a regular series of loops and nodes. However, the nodes of this

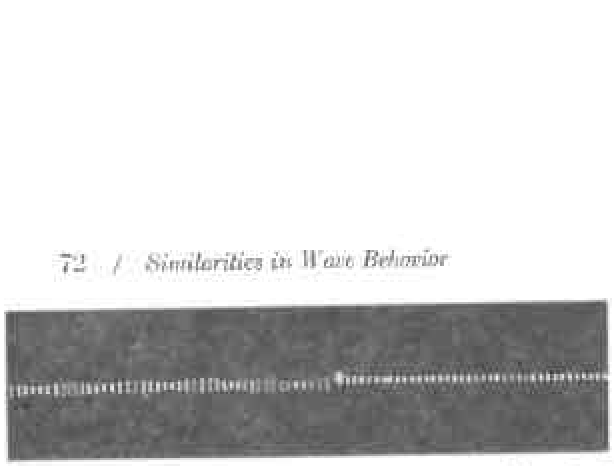

Figure 7-4. On the long-crossarm wave machine (left), a partial standing-wave envelope appears.

pattern are not completely stationary, as they were when we had a totally reflecting termination. In the present case reflection is only partial. The two wave trains passing through each other on the first machine have different amplitudes, and the pattern you see may properly be called a partial standing-wave pattern. Figure 7-4 is a time-exposure photograph of several cycles' exposure of my wave machines. This partial standing-wave pattern is clearly evident on the first one.

To understand just how this partial standing-wave pattern occurs, it may help to repeat the graphical constructions you went through back in Chapter 5 [Figures 5-4(a) through 5-4(m)]. This time I am going to let you do the constructions yourself, using wave trains A and B with amplitudes differing by a factor of two. Take your twelve resultants (the thirteenth is merely a cyclic repetition of the first), and replot all on the same horizontal axis. The picture you get should look like Figure 7-5, which shows the twelve successive appearances of the wave machine at twelve successive instants a twelfth of a wave period apart. You can see that the partial standing-wave envelope consists of a series of maxima and minima of amplitude. The minima, which correspond to the stationary nodes of the complete standing-wave pattern for a perfect reflector, are half a wave length apart

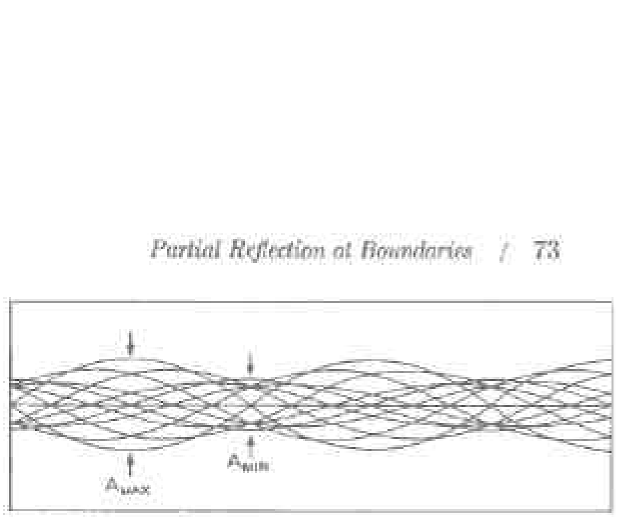

Figure 7-5. The successive "snapshots" of the medium, one-twelfth of a wave period apart, give this partial standing-wave composite for waves of same wave length and different amplitudes traveling in opposite directions.

along the machine as before, but are not loci of zero amplitude.

From the way the partial standing-wave pattern developed out of these constructions, you can see that the difference between the maximum and minimum amplitudes of the partial standing-wave envelope (A_{max} and A_{min}) will be larger as the amplitudes of the going and returning wave trains are brought closer to equality. In other words, the ratio of A_{max} to A_{min} should have some relationship to the fraction of the energy reflected by whatever impedance discontinuity is causing the reflection and producing the partial standing-wave pattern. Such a relationship does, in fact, exist. It is:

$$\text{Fraction of Energy Reflected} = \left(\frac{\dfrac{A_{max}}{A_{min}} - 1}{\dfrac{A_{max}}{A_{min}} + 1}\right)^2. \quad (7\text{-}1)$$

The ratio of A_{max}/A_{min} appears frequently in calculations of this sort. For the sake of convenience, it has been given a name — *Standing Wave Ratio*, abbreviated SWR. It is easily determined by measuring the envelope amplitudes A_{max} and A_{min} and calculating their ratio. The energy reflection equation then becomes:

$$\text{Fraction of Energy Reflected} = \left(\frac{\text{SWR} - 1}{\text{SWR} + 1}\right)^2. \quad (7\text{-}2)$$

If you ponder this equation and its meaning for a moment, you can see that it gives the correct result for two of the cases we have already studied; namely, total reflection and zero reflection. In the case of total reflection, the nodes of the standing-wave pattern are stationary; A_{min} is zero, the ratio of A_{max} to A_{min} is infinity, and the fraction reflected becomes $(\infty / \infty)^2$, which is unity, or 100 per cent reflection. On the other hand, in the case of zero reflection, there is no standing-wave pattern; A_{max} is indistinguishable from A_{min}, the SWR becomes equal to 1, and the fraction reflected works out to $(0/2)^2$, which yields zero for the result.

It may be instructive to note here that a determination, by the standing-wave method, of the fraction of wave energy reflected at an impedance discontinuity can be verified by another determination based on an expression independently developed by Lord Rayleigh. Rayleigh's expression is used to calculate the fraction of the energy in a light beam reflected at the interface between two transparent optical media such as glass and air. It goes as follows:

$$\text{Fraction of Energy Reflected} = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}, \quad (7\text{-}3)$$

where n_1 and n_2 are the optical indices of refraction of the two media. In this expression it is assumed that

the two media are optically transparent and that the beam of light is incident perpendicularly on the boundary plane between them. Remembering that the index of refraction is simply a ratio between the speed of light in vacuum, S_v, and the speed of light in the medium in question, we may rewrite equation (7-3) as:

$$\text{Fraction of Energy Reflected} = \frac{\left(\frac{S_v}{S_1} - \frac{S_v}{S_2}\right)^2}{\left(\frac{S_v}{S_1} + \frac{S_v}{S_2}\right)^2} = \left(\frac{S_2 - S_1}{S_2 + S_1}\right)^2. \quad (7\text{-}4)$$

Equation (7-4) says that all one needs to know to calculate the amount of reflection at an optical boundary is the speeds of light in the two media. If there is any truth at all in my assertion that waves of all kinds behave basically alike, then an expression which is valid for light waves should also be valid for mechanical waves on wave machines. With such an expression we should be able to calculate the amount of reflection at the joining point of our wave machines by using simple measurements of wave speeds on the two machines. These can be determined, as usual, with a meter stick and stop watch.

The subject of partial reflection will probably mean a great deal more to you if you, yourself, calculate the fraction of the energy reflected at the joining point of your two machines, both by the standing-wave ratio method and by Rayleigh's method. I consider it very important that you do these exercises. To guide you, here are data taken from actual experiments on my machines.

Standing-Wave Ratio Method

$A_{max} = 5$ mm,

$A_{min} = 1.5$ mm,

$$\text{SWR} = \frac{5}{1.5}, = 3.3,$$

$$\text{Fraction Energy Reflected} = \left(\frac{3.3 - 1}{3.3 + 1}\right)^2, = 0.29, = 29\%.$$

Rayleigh's Method

$S_1 = 15$ inches per sec,

$S_2 = 47$ inches per sec,

$$\text{Fraction Energy Reflected} = \frac{(47 - 15)^2}{(47 + 15)^2}, = 0.27, = 27\%.$$

One of the appealing features of scientific work is that when you wish to determine something, you often have to measure something entirely different and then do some calculating. To my mind, there is something earth-shakingly fascinating in being able to calculate a reflection coefficient on a *mechanical* wave system by either of two different methods, one having evolved from the field of *electrical* engineering, and the other from the field of *optical* physics — *with neither method involving a direct measurement of the thing actually being sought*!

If you are skeptical and insist on a direct measurement of the fraction of the wave energy reflected at the junction point of your two machines, you can make this measurement by comparing the amplitudes of the incident and partly reflected pulse in an experiment similar to the one illustrated in Figure 7–3. Since the energy of a wave is proportional to the square of its amplitude, the fraction of the energy reflected will be the square of the ratio of the amplitude of a partially reflected pulse to the amplitude of the incident pulse

which produces it. By direct measurement from pulses A and B (Figure 7–3), I obtain an energy reflection of 21 per cent. If I had allowed for the fact that pulse A was a little lower in amplitude by the time it had traveled to the point of reflection from where it is shown in the photograph; and if I had allowed for the fact that the partially reflected pulse B had also died out a little by the time it traveled back to where it is in the photograph, my direct measurement result would undoubtedly agree even more than it does with those calculated by the standing-wave ratio method and by the Rayleigh method.

An investigation of the location of the nodes in the standing-wave pattern for different terminating im-

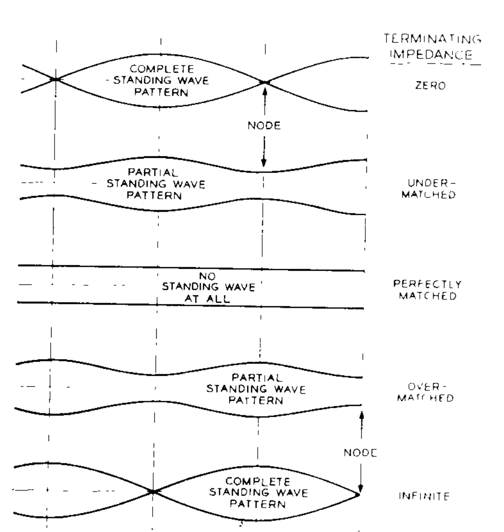

Figure 7-6. As the impedance terminating a wave medium is increased through the matching value, the location of the nearest node of displacement moves one quarter wave length toward the termination.

pedances may interest the more advanced investigator. We have seen that, for a zero-impedance termination on the end of a wave medium, the nearest node of displacement is a quarter wave length away, while for an infinite-impedance termination, the termination itself is a node, with its closest neighbor one-half a wave length back along the medium.

By investigating the pattern of partial standing waves produced by terminating your wave machine with the dash pot at different positions along the last crossarm, including both sides of the matching position, you can verify that the pattern of nodes moves one quarter wave length toward the end of the medium as you go through the matching condition.

A sequence of such patterns is shown in Figure 7-6. Here the standing wave and partial standing-wave envelopes are sketched for different terminating impedance conditions. It will be difficult to locate exactly where the nodes come on the wave machine for impedances close to the matching value — that is, for which the SWR's are close to unity. However, you shouldn't have any difficulty with impedance values sufficiently disparate from the matching value to give SWR's of 2 or more.

CHAPTER EIGHT

Transformers

In modern technology it is often necessary to transmit wave energy from one propagation medium to another of a different impedance. Thus, light is made to pass from air to glass and back to air again. Sometimes the transition is repeated several times — as in the optical system of a binocular or in a camera lens having several individual components.

Electric power is usually generated at low impedance and then, for reasons of economy, transmitted over high-impedance transmission lines to its destination. Here it is transferred to low-impedance local distribution networks for customer use. In microwave communication systems, high-frequency electromagnetic waves, traveling in waveguides, must be launched as radio beams into a medium of different impedance — air.

Whenever wave energy must be transferred from one medium to another of a different impedance, partial reflection normally takes place. However, partial reflection is economically wasteful, since that portion of energy which is reflected never reaches the other end of the system. The question is, "How can you transmit wave energy from one medium to another, across an impedance discontinuity, *without* suffering reflection losses?"

Transformers provide the solution. A transformer is a device which is inserted into the propagation path at the point where impedance changes abruptly. It has the effect of smoothing over the discontinuity so that reflection is minimized or prevented altogether. In short, transformers allow us to accomplish reflectionless transfer of energy.

The simplest type of transformer is one which, when inserted between two media having different impedances, becomes itself a wave medium with a gradual impedance taper along its length. At its two ends it has impedances matching the impedances of the media to be joined. Between the ends the impedance tapers from high to low, or vice versa — depending on which way you are looking toward the transformer.

An approximate example of such a transformer in the field of acoustics is a cheer leader's megaphone. The air column in a person's throat and mouth has an impedance quite different from that of the free air in front of his face. Ordinarily, speech is very inefficient because much of the sound power which the vocal cords are potentially able to deliver is reflected back into the throat by the impedance discontinuity at the lips. We can smooth over this discontinuity with the tapered air column in a megaphone. The air column at the small end has an impedance approximating that of the human throat and mouth. At the large end the impedance approximates that of free air, thus producing an over-all impedance match which results in less reflection and greater output.

One advantage of using a megaphone is that it concentrates the sound in a forward direction by reducing its lateral dispersal. Another is that, by providing an impedance match, it enables you to put out more sound power than you could without its help.

A Taper Transformer for Your Wave Machines

You can easily make a taper transformer section to sandwich in between your two wave machines and see for yourself how its presence reduces the reflection of waves and pulses.

Proceed as suggested in the sketch of Figure 8-1, with a 10- or 12-inch section of wave medium. Use crossarms of the same diameter, spaced the same distance apart center-to-center along the backbone as on your two wave machines, but with each crossarm shorter by the same amount than the one preceding it. The first crossarm of your transformer section should have the same length as the crossarms of your first wave machine; the last crossarm should have the same length as those of your second wave machine. The transformer section may be supported in bearings similar to those used for your two main machines, and may be joined to these machines by the same kind of clamping arrangement previously described. Figure 8-2 is a photograph of my two wave machines joined by this kind of taper transformer.

Now launch a short, quick pulse on the input end of the first machine. Watch it travel along and transfer across the taper-section transformer onto the second

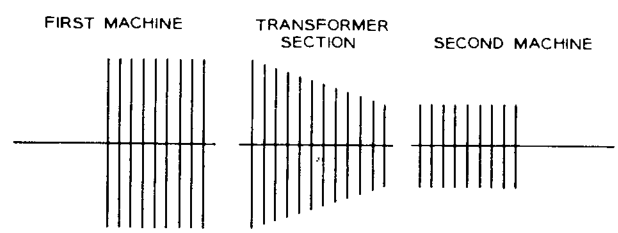

Figure 8-1. This taper-section transformer is the mechanical analog of a megaphone or an ear trumpet.

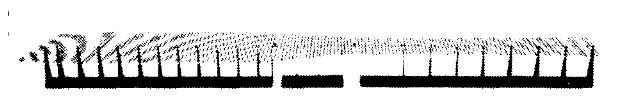

Figure 8-2. My assembled wave system, with taper section transformer in place, looks like this.

machine. Note the small amount of reflection at the transformer - - much less than there would have been without it. The transformer has indeed smoothed over the impedance discontinuity and permitted a more effective transmission of wave energy to the second machine.

You can verify this improved transmission with a similar experiment. This time use continuous waves, generated by the motor-and-crank generator. Place your dash pot at the far end of the second wave machine in a matching position. Now generate continuous waves with the motor-and-crank attachment and measure the SWR on the first machine, just ahead of the transformer. You'll find the SWR is much closer to unity than when you had the two machines coupled without the transformer. In fact, you may have trouble locating the loops and nodes of the partial standing wave, so small will be the difference between A_{max} and A_{min}. Make the comparison and see for yourself. What percentage reflection do you find? What was it before, without the transformer?

Step-up and Step-down Action of Transformers

If you learned about electrical transformers in an elementary physics course, they were probably presented to you as devices for stepping up or stepping

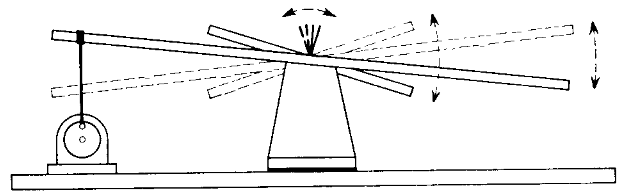

Figure 8-3. As waves travel from the long-crossarm machine to the short-crossarm machine the taper transformer steps down the torque and steps up the angular amplitude of the oscillation.

down the voltages applied to their primaries. You learned that if an ideal transformer is used to step down a voltage, it will step up the current in the same ratio.

In analogous fashion, the mechanical taper transformer gives a step down in torque and a step up in angular velocity and angular displacement by the same factor. You can easily see this step up in angular displacement if you look end-on down the central wire of your wave machine. The oscillating angular arc swept out by the short crossarms of the second wave machine is perceptibly larger than the arc executed by the long crossarms of the first machine. Figure 8-3 sketches what you'll see when you look down the axis of your mechanical system this way.

To wax quantitative for a moment, consider an ideal electrical transformer* having 100 turns in its primary coil and 50 in its secondary. With 100 volts across its primary, the transformer will deliver 50 volts at the secondary terminals. And, if a current of one ampere is drawn in the secondary circuit, a current of only one-half ampere will be required in the primary.

* **Special Note for Fussy Perfectionists** — All these observations and deductions assume that you are dealing with an ideal

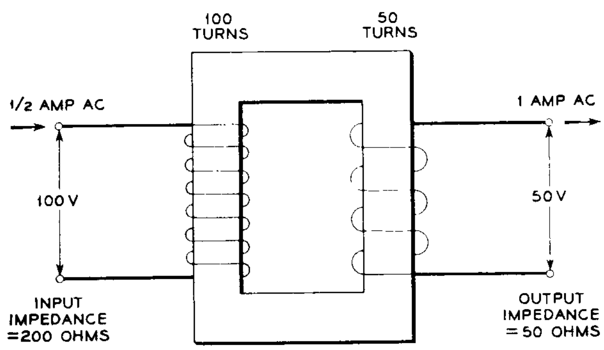

Figure 8-4. The impedance ratio of an ideal electrical transformer is the *square* of the voltage ratio.

These relationships are illustrated in Figure 8-4. Under these conditions, the transformer is operating at an input impedance of 100/0.5, or 200 ohms; and an output impedance of 50/1.0, or 50 ohms. The input-output impedance ratio is 4 to 1. In other words, the impedance ratio is the *square* of either the voltage step down ratio or the current step up ratio. By analogy, you may infer that the mechanical impedance ratio of your two wave machines is the square of the angular displacement ratio you observed in the experiment described above.

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transformer; i.e., one which achieves perfect matching between the wave media it connects, gives no reflection whatsoever, and consumes no energy within itself. The taper transformer you built according to Figure 8-1 is not ideal. To be sure, it reduces the reflection occurring when a wave is transferred from the first machine to the second. It does not, however, eliminate it altogether. You can make a more ideal transformer, if you wish, by feathering off the taper of the transformer structure at the two ends of the transformer section (as shown in Figure 8-5) instead of using the linear taper of Figure 8-1.

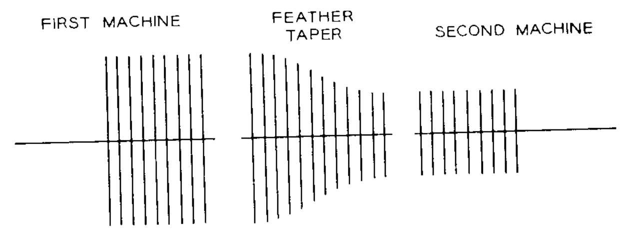


Figure 8-5. By "feathering" the ends of the taper you can make a more nearly ideal taper transformer.

Theory of the Taper Transformer

The physical theory of a taper transformer can be described in terms of the operation of a conical taper section joining two cylindrical acoustic tubes of different diameters, as illustrated in Figure 8-6. The taper of the conical section may be regarded as the physical limit of a very large number of very small steps between very short cylindrical sections of progressively smaller diameter. At each step a very small part of the sound energy is reflected. However, the part reflected at step A is cancelled by the part reflected by step A′, one quarter wave length farther on, which returns to A just one half of a wave period later. Similarly, the partial reflections from step B are cancelled by destructive superposition of reflections from step B′, and so on. This cancellation of the partial reflections from different parts of the transformer results in a reduction of the total net reflection. If you can make the taper half a wave length or more in length and properly shaped, you will have an ideal transformer. It will give no reflection at all. Such a transformer, like the electrical one I just mentioned, works equally well for waves traveling in either direction.

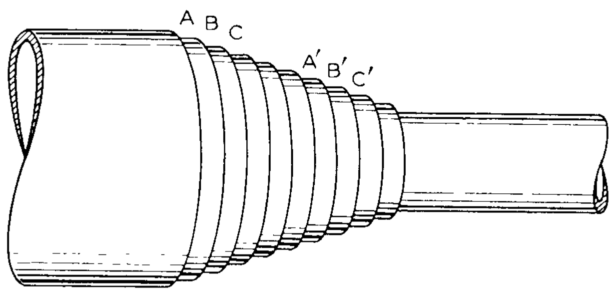


Figure 8-6. A taper transformer may be regarded as the limit of a large number of small, slightly reflecting steps.

The Quarter-wave Transformer

After reading that a taper transformer works by the interference of reflected waves at various zones along its length, you may now be thinking, "Why not eliminate the taper altogether and reduce the transformer to two reflecting steps, a quarter of a wave length apart? Then the partial reflection from the second step would cancel that from the first, giving no net reflection at all."

This is a splendid idea, and I hope it really did occur to you. Such a transformer works to perfection. Theory tells us that the impedance of the transformer section between the two steps should be the geometric mean of the impedances of the two media you are connecting to the transformer, and that the two steps should, indeed, be a quarter wave length apart.

A quarter-wave transformer section for use with your two wave machines would look something like the one sketched in Figure 8-7. To make one which best fits *your* situation you will have to compute how long the section should be, and also how long to make the crossarms. I'm going to let you compute these quantities for yourself.

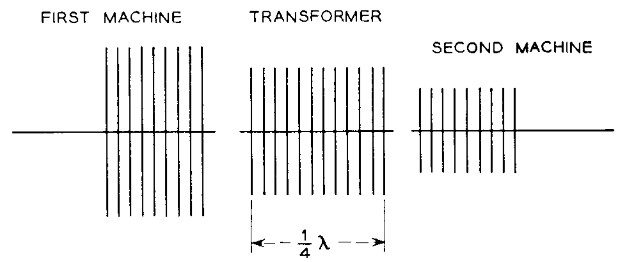


Figure 8-7. This quarter-wave transformer does in two steps what the taper transformer does in an infinite number of steps.

Assuming that your two wave machines and the proposed transformer will have central wires of the same diameter and material, the impedance of each section will be proportional to the square root of the rotational inertia of the crossarm system per unit length (equation 3–3). If you know the frequency of your wave generator and the speed of the waves in your transformer section (these can be computed from equation 3–1), you can determine how long the section should be made in order to have a quarter-wave length. The quarter wave length, by the way, refers to a quarter wave length *in the transformer medium*, not in either of the two machines.

After having built your transformer section according to the above considerations and mounted it in bearings, sandwich it in between your two wave machines and see how effective it is in cutting down reflection losses. To do this, measure the SWR on the first wave machine, first with the transformer section in place, and then with the two machines directly connected without the transformer. Calculate and compare the fraction of energy reflected in both cases. In these measurements, the dash pot should be attached to the output end of the second machine in a matching position.

You can imagine, knowing how a quarter-wave-length transformer operates by mutual cancellation of the two reflections, that such a transformer will not work very well for single waves and pulses. You can verify this conclusion by experiment. A quarter-wave-length transformer is effective only with continuous waves. Moreover, it is effective only with continuous waves at and near the particular frequency for which the transformer was designed. The taper transformer, on the other hand, works fairly well with single waves and pulses, as well as with continuous waves. And it does so over a broad range of frequencies. It is subject only to the limitation that the half wave length of the waves must be shorter than the length of the taper section.

Practical Uses of Quarter-wave and Taper Transformers

By now it must be apparent to you why taper and quarter-wave transformers aren't used on power line systems. At a frequency of sixty cycles per second (depending, of course, on the speed of wave propagation along the lines in question), wave lengths may be hundreds of miles. A transformer of a quarter of this length would be rather awkward, to put it mildly. Consequently, other means of impedance transformation are resorted to. The familiar assemblies of primary and secondary coils of wire on iron cores are the result.

The story is very different when we come to the realm of microwave propagation employing very high frequencies and extremely short wave lengths. In these frequency ranges wire coil transformers cannot be used because of the inductance of the windings. Taper sections and quarter-wave sections are employed

Figure 8-8. This taper transformer section of rectangular waveguide is a megaphone for microwaves.

almost exclusively for impedance transformation in these frequency ranges. Figure 8-8 is a photograph of a taper-section transformer used to couple a rectangular waveguide to another rectangular waveguide of different cross-section dimensions. It looks like a rectangular megaphone. That's exactly what it is, too, except that it propagates electromagnetic rather than acoustic waves.

Quarter-wave transformers are quite familiar to you in the field of applied optics. The non-reflecting coatings on camera and binocular lenses are quarter-wave transformers designed to match the impedance of air to that of glass at optical frequencies, thus eliminating bothersome reflections. A non-reflecting coating on a glass lens is simply a film, a quarter wave length thick, of some transparent material whose index of refraction is geometrically intermediate between that of air and that of glass.

Nature, too, has developed some interesting transformers in her course of evolution. Inside the mammalian ear are three tiny bones called the hammer, anvil, and stirrup. Their function is to provide a mechanical impedance linkage between the low-impedance air column in the outer ear and the higher-impedance liquid column in the inner ear. We find the same sort of thing in the various bone, joint, and ligament arrangements of animals. These are natural impedance-matching transformers designed to utilize most efficiently the energy available from muscles. Compare the long bones and short distances from the joint to the point of attachment of the muscles in a swift-running animal, such as the gazelle, with the short, heavy bones and relatively long distances from the joints to the point of attachment of the muscles

in a lumbering animal, such as the alligator or giant tortoise. In the case of the gazelle, the object is to carry a light load at high speed. Alligators, on the other hand, are built to carry a heavy load at low speed.

In technology, as in nature, impedance matching has come to be almost a way of life. This is true even in cases where wave propagation is not involved, and where action is unidirectional, rather than oscillatory. Propulsion power for a ship is developed in a high-speed, low-torque (low-impedance) turbine. The power must be applied to a low-speed, high-torque (high-impedance) propeller shaft. Direct coupling of the turbine shaft to the propeller would stall the turbine. To match the impedances of the generator and the load, a gear train is introduced as a transformer to provide an optimum transfer of power from turbine to propeller. Such a transformer provides a step down of angular velocity and a step up of torque. Gear trains, levers, and all the other mechanical-advantage machines you learned about in high school physics are really, as you will recognize when you think about them, impedance transforming devices.

CHAPTER NINE

Loaded Lines, Interferometry, and Wave Filters

Up to now we've discussed only wave propagation systems having uniform properties at all points except those at which the impedance changes abruptly — as at a boundary between two different media. We have also experimentally verified that wave speeds are constant in a uniform elastic medium, regardless of the amplitude or length of the waves.

However, when we deal with wave media which are not uniform, or in which non-uniformities are periodically spaced along the propagation path, we find a new family of manifestations resulting from the interference of partly reflected waves. These manifestations are classified under the general heading of interferometry.

Setting Up for Interferometry Experiments

To do the experiments described in this chapter you will need some equipment not yet called for in any of the previous demonstrations. In particular, you will need some means of generating waves at many different frequencies and wave lengths. If the motor you have been using is a synchronous one, such as a geared-down clock motor, and if you have access to a variable-frequency oscillator with a range of from 20 to 200 cycles per second and sufficient output to drive the motor, you will have no problem. Lacking such equip-

ment, you can rig up a variable-frequency oscillating drive for your wave machine by using a pendulum as a generator. Such an arrangement is shown in Figure 9-1.

As the pendulum swings, it gives an oscillating rotary motion to the pivot wire. This pivot wire can be clamped to the input end of your longer crossarm wave machine with the same clamping arrangement you used to couple the wave machines to each other in earlier experiments.

The pendulum mass should be quite large, ten

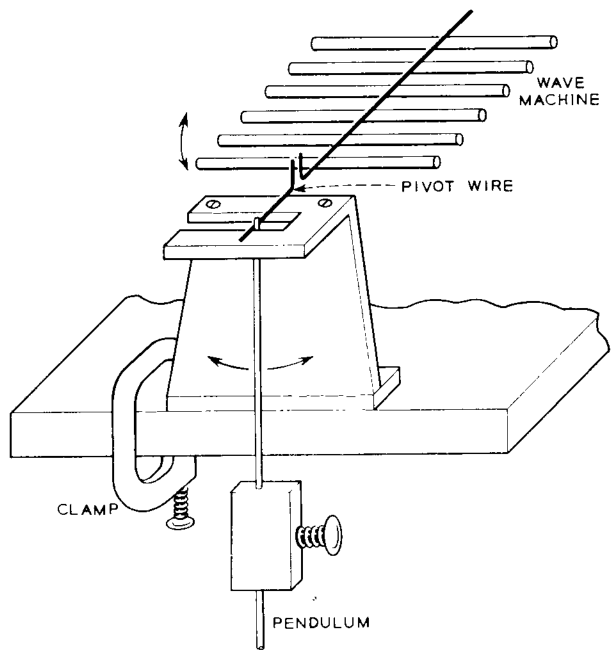


Figure 9-1. With this additional equipment you have a pendulum wave generator whose frequency you can vary at will.

pounds or more, in order that the swinging of the pendulum will not be appreciably affected by feedback from the wave machine. This mass must be provided with a clamping arrangement for raising or lowering it along the pendulum rod, so as to give an oscillation at any desired frequency over a considerable range. With a little practice, you will find that you can keep this pendulum swinging at very nearly constant amplitude by slightly pushing the rod sideways each time the mass passes its midpoint. With every cycle of its swing, this pendulum generates one cycle of a continuous train of waves on the wave machine.

You will also have to make provisions for placing extra mass on the ends of certain of the crossarms of the longer crossarm wave machine. If you have access to a machine shop, you can make rectangular brass or iron weights that can be slid onto the ends of the crossarms and clamped in place by set screws, as suggested in Figure 9-2. Alternatively, you can use lead fish-line sinkers, screwed or soldered to peewee

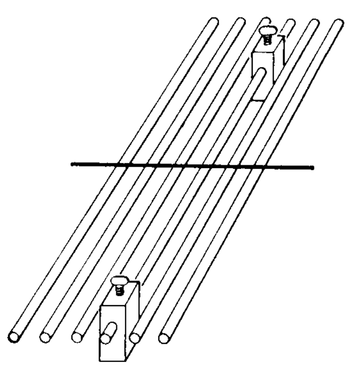


Figure 9-2. Added mass can be added to certain crossarms by means of weights like these.

battery clips. These, in turn, can be clipped to the ends of the crossarms. Weights of equal mass should be attached to opposite ends of the crossarms to be loaded, so as to maintain the balance of the wave machine in its bearings. The combined mass of the two weights attached to a mass-loaded crossarm should have a total mass approximately equal to that of the crossarm itself. You will need about a dozen of these weights.

Now attach a pair of these weights to the opposite ends of a crossarm near the middle of your machine. This additional load creates an impedance discontinuity in the propagation path, as you can verify by sending a pulse down the machine and observing the partial reflection which occurs at the loaded crossarm. If you attach another pair of weights to the ends of another crossarm, five or six inches farther on down the machine, you will have two impedance discontinuities in the transmission path. Each discontinuity produces partial reflection of a wave or pulse traveling down the machine. You may be able to spot these two reflections if you launch a short pulse and watch what happens to it as it traverses the two loaded crossarms. Try it.

Now attach your continuous-wave generator to the input crossarm of the machine. Adjust the frequency so that the wave length of the waves on the machine is about half the length of the machine itself. Attach the impedance dash pot *in a matching position* to the last crossarm. Measure and record the ratio of wave amplitude at the dash pot to the amplitude at the generator. The square of this ratio will be the fraction of the input energy which is transmitted to the output. We may call it the transmission coefficient. Now

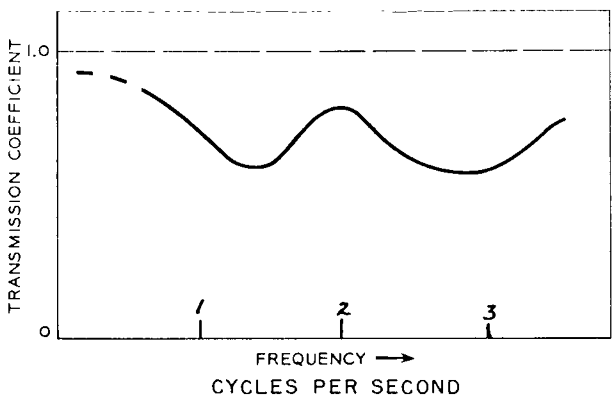


Figure 9-3. This frequency characteristic was obtained for two loaded crossarms on my wave machine.

increase the frequency somewhat and reduce the wave length on the machine. You will, in general, find that the transmission coefficient has also changed In small steps, progressively increase the frequency up to the limit of the frequency range you have available, measuring the transmission coefficient at each step. When you plot your data — transmission coefficient as a function of frequency — you will find a transmission characteristic like the one shown in Figure 9-3.

Why the dips and humps? For an answer, I suggest you go back to the frequency corresponding to the bottom of the first dip, and measure the wave length on the machine of the waves having this frequency. What relationship does this wave length have to the distance between the two loaded crossarms? "Exactly twice," you say. Fine, but before reading farther, *you* figure out what is happening to make transmission a minimum at this frequency.

A Slight Case of Constructive Superposition

Have you figured it out? Did you deduce that, when the wave length is exactly twice the distance between the two partial reflectors, the wave portion reflected at the second loaded crossarm travels back to the first loaded crossarm with just the proper timing to coincide, crest-on-crest, with the portion of the next wave reflected at the first loaded crossarm?

What's happening is that the two reflections are reinforcing each other by constructive superposition at this frequency. Hence, more wave energy is reflected than at any neighboring frequency, for which the two reflections do not add up in step but partly cancel each other instead. The frequency at which the reflection is a maximum is, of course, the one at which the transmission is a minimum. Another minimum is found at twice the frequency of the first. At this frequency, the wave length equals the distance between the partial reflectors and the phase relationships are again just right for constructive superposition of the reflected waves and maximization of the reflected energy.

The arrangement you've set up for this experiment is the mechanical analog of an optical device called a Fabry-Perot interferometer. The optical interferometer consists of a pair of half-silvered (half-reflecting) glass surfaces, aligned parallel with each other and oriented perpendicular to a beam of light which shines through them. If the emergent beam is examined with a spectroscope, the continuous spectrum of light from a white source is seen to be "channeled" by regions which appear less intense then neighboring regions. These low-intensity regions correspond to optical frequencies for which the separation of the half-silvered surfaces is an exact number of half wave lengths of the light. When one moves the half-silvered surfaces close together or farther apart, the dark regions will be seen either moving away from each other or crowding closer together in the field of view of the spectroscope.

Frequency Filtering

Now add weights to the ends of two more crossarms, one on the generator side of the original loaded pair and the other on the output side. Space these so you end up with the four loaded crossarms at equal distances along your wave machine. Repeat the series of measurements you made before and draw a new plot — transmission coefficient as a function of frequency. In this new plot, you'll find the dips are deeper and steeper-sided than in the first one. This is understandable; with four reflectors, you *should* get more reflection at frequencies for which the reflected portions of the waves superpose, crest-on-crest. Note, however, that the dips come at the *same frequencies* as before.

Repeat the experiment now with a still further modification. This time, add weights to the ends of still more crossarms, again spacing them at equal intervals. And, once more, make the transmission measurements at the various frequencies and plot the transmission curve. You'll find the dips are very steep-sided and very deep — practically down to the axis of zero transmission. The knees of the transmission curve are almost right-angled. The wave machine has become a very frequency-dependent thing; whereas, before any of the crossarms were loaded, it transmitted all frequencies with equal facility. It is now an efficient filter structure. The ranges of frequency which can be

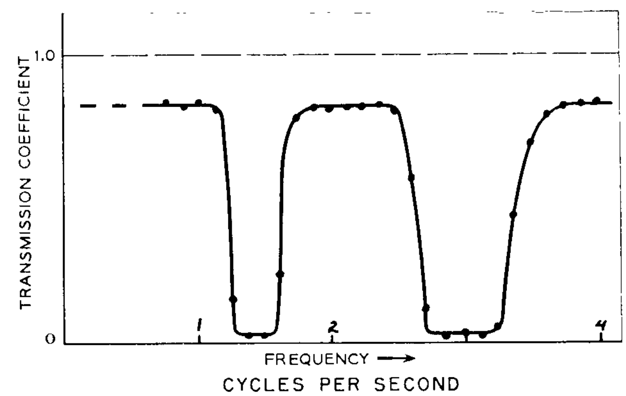


Figure 9-4. These actual data were taken with my wave machine loaded with extra mass at every tenth crossarm.

readily transmitted are called *pass bands;* those which are not transmitted are called *stop bands* (see Figure 9-4).

Set your generator at a frequency close to the middle of the first stop band and then stand off and look at the wave pattern on the machine. You can expect some kind of standing wave, because you now have original waves going in one direction and reflected waves re-

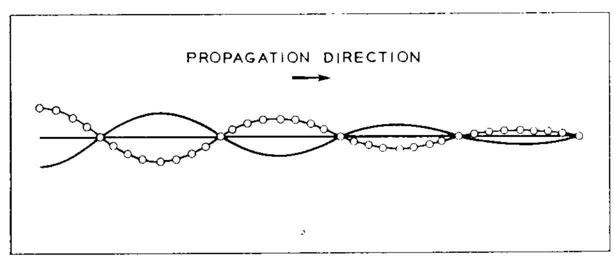


Figure 9-5. In the stop band of a periodically loaded structure the amplitude of a wave decreases rapidly from section to section.

turning. But the returning waves, instead of being reflected all at once by a single total reflector, are reflected piecemeal by several partial reflectors. Thus, the standing-wave amplitude diminishes as you go from the generator toward the output end of the machine. Figure 9-5 is a sketch of this pattern.

Practical Uses of Filter Structures

Filter structures are of great practical importance in electrical, mechanical, acoustic, and optical engineering. In carrier telephony, for example, it is often necessary to segregate a particular frequency range from the continuous frequency spectrum in which it lies. Such a frequency band might be a speech "channel" about 3500 cycles wide—just wide enough to include most of the frequencies used in human speech. Or the desired band may have a much broader range of frequencies—say, 3,000,000 to 5,000,000 cycles, enough to accommodate a standard TV channel.

In acoustics, too, it is often desired to enhance or suppress certain audible frequency ranges. And in mechanics it is frequently necessary to shock-mount a delicate piece of apparatus to protect it from jarring and vibration. Isolating the passengers in an automobile from the mechanical vibrations of a rough road is a case in point.

In all the varied examples cited above, the method of frequency filtering is basically the same. The engineering design calls for the deliberate introduction into the transmission path of impedance discontinuities. These discontinuities, by causing the reflection and interference of waves, selectively suppress or enhance their propagation in various frequency ranges.

CHAPTER TEN

Wave Analogies

We have thus far concerned ourselves almost exclusively with the mechanics of wave behavior. In this chapter I'd like to serve up a little food for philosophical thought about the universality of many phenomena of nature. In previous chapters analogies were freely used to show how predictable and similar is the behavior of waves throughout the various fields of physics. I now mean to use the analogy method to demonstrate and discuss more complicated and esoteric phenomena.

Consider, for example, the old turn-of-the-century acoustic demonstration, which can easily be patched together with a couple of discarded mailing tubes. An input tube (Figure 10-1) delivers sound to two branches, one longer than the other. The branches come together again in an output tube. If the incoming sound is of such a frequency that one branch is half a wave length longer than the other, destructive interference occurs at the rejoining point and no sound comes out of the tail pipe. Under these conditions standing waves are set up in the branches — standing waves having a node at the output junction. You get no sound from the tail pipe because you can't get energy out of a standing wave system by coupling into it at a node.

The electrical analog of this acoustic situation is a transmission line terminated by a parallel inductance-capacitance arrangement (Figure 10-2). If electrical waves are coming along the line, having the same

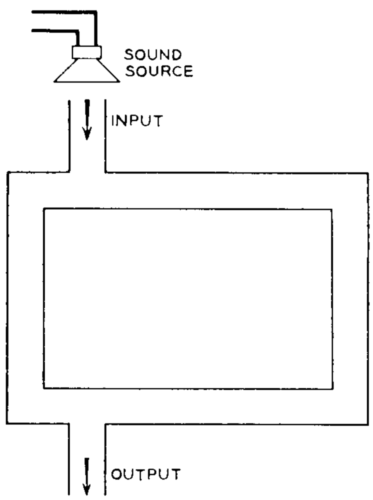


Figure 10-1. If one branch is half a wave length longer than the other no sound comes out at the output.

frequency as the natural frequency of the LC branch, then nodes of voltage appear between the plates of the capacitor and at the center tap of the inductor, while nodes of current occur at the joining points of the branch circuit with the main line. Even though enor-

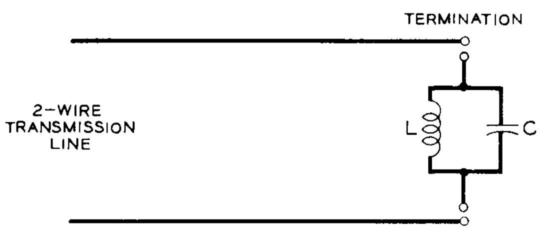


Figure 10-2. This LC branched termination on the end of a transmission line is an electrical equivalent of the branched acoustic tube.

mous currents may slosh back and forth between the plates of the capacitor and through the coil, no ac current flows through the parallel termination from the main line at this frequency. This termination looks like an open-circuited end to the main line.

These two analogous situations have a mechanical analog which can be demonstrated with your two wave machines. To set up for this demonstration, you will need to mount the short crossarm machine above the long crossarm machine. It should slightly overlap it in length, as shown in Figure 10-3. This mounting can be done with the aid of two three-legged platforms. Build these to straddle the longer crossarm machine, as shown in Figure 10-4.

From past experiments you should have a good idea of the wave lengths your motor-and-eccentric-crank generator will produce on your two machines. By means of two light, stiff connecting wires attached to the ends of the crossarms, connect the two machines together in such a way that the first quarter-wave length on the upper machine is connected across the last three-quarters of a wave length on the lower machine. Since the wave speeds and wave lengths on the two machines are different, the actual distances on the machines between connecting wires may be nearly equal.

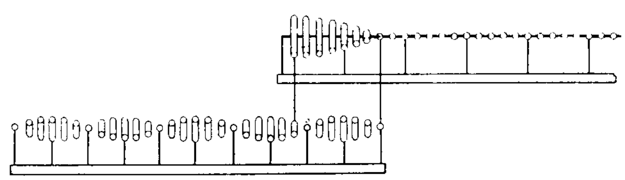


Figure 10-3. This arrangement and cross-connecting of the wave machines gives a mechanical analog of the branched acoustic tube of Figure 10-1.

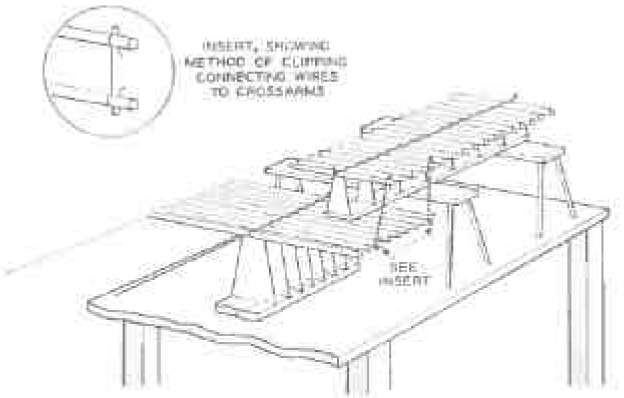


Figure 10-4. Two light platforms can be used as shown to mount the short-crossarm machine over the long-crossarm machine in the proper relationship for cross-connecting with stiff wires.

Now connect the generator to the input end of the lower machine and start it running. After the starting transient has disappeared, see what amplitude of wave you have on the output section of the upper machine. Now use different connecting wire arrangements by moving the connecting clips from one crossarm to the next in both directions of both machines. Try to find a connecting arrangement which reduces the output wave amplitude on the upper machine. Finally, zero in on an arrangement which gives no output at all on the tail end of the upper machine.* Figure 10-5 is a time-exposure photograph of my two machines connected to produce such a cancellation of the output.

* In general, the proper cross-connecting points will not come exactly at one-quarter wave length on the upper machine and three-quarters wave length on the lower machine. The two machines have different impedances, and the location of the exact cross-connecting points will depend also upon how far out on the crossarms you attach the connecting wires.

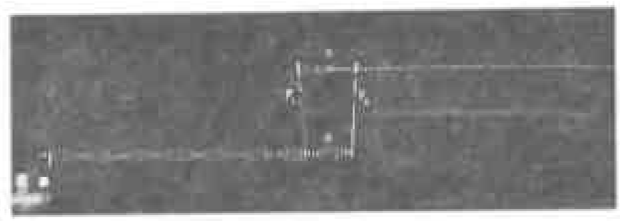


Figure 10-5. This time-exposure photograph shows the interference which produces no output on the right-hand section of the upper wave machine.

Perhaps now you can see what is going on here. At the first branching point, P_1, waves of equal amplitude and frequency are transferred onto the two branch sections A and B. The waves travel with different speeds along these two branches and arrive at the rejoining point, P_2, half a cycle out of phase with each other, thus producing cancellation by superposition at this point. The rejoining point thus becomes a node of the standing-wave patterns on both branches; and, since the output section is connected into the branch system at this point, no output appears here. I repeat: No energy can be taken out of a standing-wave system by coupling into it at a node.

Observe that a standing wave also appears on the input section of the lower wave machine. This standing wave has stationary nodes, indicating that the branch arrangement behaves like a perfect reflector. That is, it transmits no energy and absorbs no energy.

Low-pass Filters

In the last chapter we discussed some of the physics of repeated-section frequency-sensitive structures. Now let's focus our attention on a particular class of such structures called low-pass filters. The frequency characteristic of a low-pass filter shows good transmission of waves of all frequencies below a certain critical frequency, and good rejection of waves of all frequencies above it. This critical frequency is determined by the parameters of the structure. Figure 10-6 illustrates these characteristics.

In electrical engineering, low-pass filters are used to smooth out the unidirectional but pulsating current delivered by an ac/dc rectifier, so as to give a steady direct current at the output of the filter. Such a filter can be built with an inductor and capacitor in a repetitive series connection as shown here:

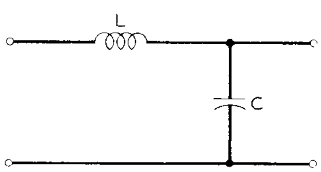


Figure 10-7 shows a three-stage low-pass filter made in this way. If the critical frequency of the filter is designed to come well below the pulsation frequency of the current delivered from the rectifier, the current flowing through the load will be smooth.

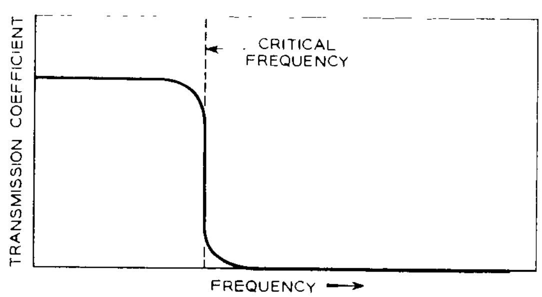


Figure 10-6. This is the frequency characteristic of a low-pass filter.

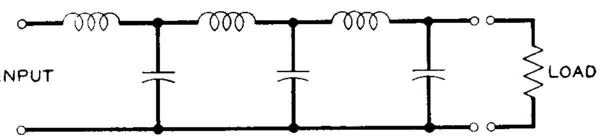
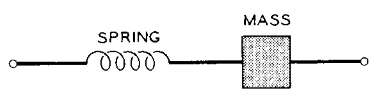


Figure 10-7. This 3-stage low-pass smoothing filter takes a pulsating input and delivers a smooth output to the load.

Mechanical Smoothing Filters

A mechanical analog of the electrical ripple-filter is the spring suspension in an automobile (Figure 10-8). Here we have a three-stage mechanical structure which prevents the up-and-down vibrations of rough roads from getting through to the rider. This filter is built up from a basic unit consisting of a spring in series with a mass, thus:



Here the spring corresponds to the shunt capacitor of the electrical filter, and the mass corresponds to the series inductor. Though the three sections of an automobile's suspension system are not identical in their spring and mass parameters, you must admit the efficacy of the combination. Automobile passengers feel relatively little vibration even when the car is driven down a corduroy road. The spring-and-mass filter is designed so that its critical frequency is well below the frequency of most rough road vibrations.

Hydraulic Smoothing Filters

The hydraulic analog of such a smoothing filter is used to smooth out the jerky flow of water entering a pipe system from a piston pump (Figure 10-9). This

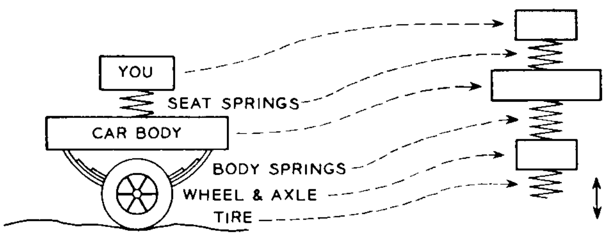
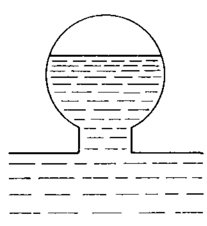


Figure 10-8. The mechanical smoothing of a rough side is accomplished by a 3-stage filter of springs and masses.

filter is made up of one or more basic stages, each consisting of a side chamber partly filled with air and a length of pipe, thus:



The air in the side chamber provides a compressible cushion for absorbing fluctuations in the flow of water.

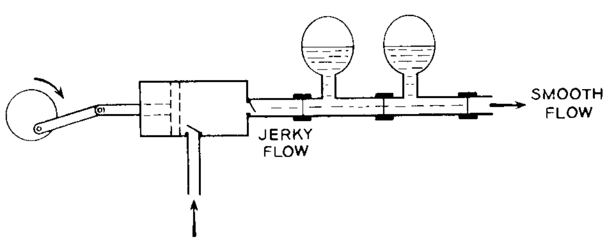


Figure 10-9. This ripple filter for a piston pump is the hydraulic analog of the electrical filter and the car spring system.

The compressibility of this cushion corresponds to the capacitance of the condenser in electrical filters, and to the springiness of the spring in mechanical filters. The inertia of moving water in the main tube corresponds to the inductance of the coil in electrical filters and to the inertia of the mass in the mechanical filters. The incoming jerky flow of water from the pump is made more steady at the output of the last stage of the filter by an amount depending upon the constants of the filter, the number of stages, and the impedance of the rest of the pipe system.

"Elastic-wall" Smoothing Filters

An interesting variant of the low-pass hydraulic filter is the heart and its associated blood vessels. The flow of blood from the left ventricle of the heart into the input end of the aortic artery is very jerky. In fact, it stops entirely during the diastolic portion of the heartbeat cycle. In this case, however, the smoothing action on the flow of blood comes not from pneumatically-cushioned side chambers, but rather from

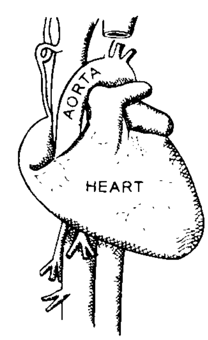


Figure 10-10. The aorta is a distributed-parameter smoothing filter.

the elasticity of the walls of the aorta itself. As each successive measure of blood is pumped into the aorta, its walls expand to accommodate the additional volume. Then, during the "off" portion of the cycle, the walls contract again, squeezing the blood onward in a nearly steady flow. The elasticity of the aorta and main arteries of the body is such that, by the time blood has traveled from the heart to the wrist or ankle, there is barely enough irregularity remaining in the flow to permit feeling of the pulse.

The elastic-wall type of smoothing filter differs from the ones considered earlier in that the elements doing the smoothing are distributed along the entire length of the propagation medium instead of being concentrated in discrete units such as capacitors, inductors, springs, masses, side chambers, etc. The elastic-wall smoothing filter is therefore classed as a distributed-parameter structure, while the others are called lumped-constant structures.

Other propagation media of the distributed-parameter variety which are analogous to each other are (1) submarine telegraph cables carrying electrical pulses representing dots and dashes, (2) the earth, into which daily and yearly temperature fluctuations penetrate, and (3) the base layers of transistors, across which fluctuating concentrations of minority carriers are diffusing.

As you see, structures exhibiting analogous wave behavior can vary enormously in size. Submarine telegraph cables may be over 3000 miles long. The base layer of a transistor may be only 10^{-4} centimeters thick! Quite a difference — yet, as propagation media for their respective types of wave, they have much in common.

CHAPTER ELEVEN

Summing Up

The examples just considered, along with those developed in earlier chapters, should once again impress you with the way so many apparently unrelated physical phenomena turn out to be quite similar after all — once we have stripped away their disguises and revealed their basic essences.

The important thing to note is that waves of *all kinds* behave fundamentally alike as they propagate, reflect, superpose, interfere, and go through their various other paces. Be they mechanical, acoustical, electrical, thermal, optical, or electromagnetic waves, they are basically sisters under the skin. If we learn to understand one, we can understand all.

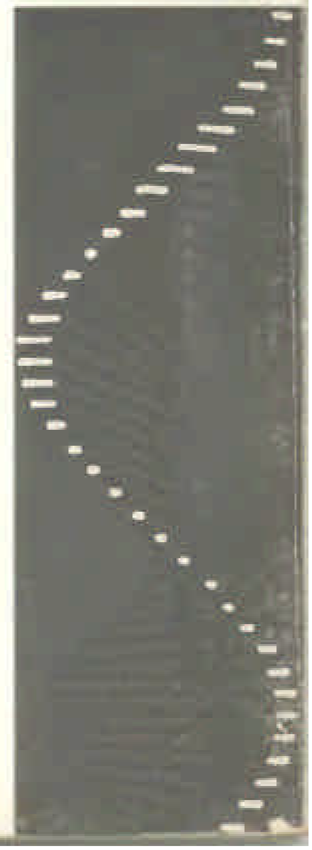
It pays to keep in mind, when undertaking the study of any new discipline, that at first sight Nature often appears arbitrary and capricious. A closer look, however, will usually reveal basic laws which govern her behavior — laws which are the same in physics, chemistry, biology, geology, astronomy, oceanography, and all the other branches of natural science. Contemplation of these basic laws should convince any skeptic of the tremendous beauty, unity, and universality of Nature, however complicated she may seem superfically to be.

If at the end of your reading of this book I were to ask you what you have learned from doing so, and if you were to reply, "I learned some interesting things about the behavior of waves," I would feel that your experience had brought you some small value for the effort you have expended. But if I were to press you, "Is that all you learned?", and if you were to reply, "Yes, I guess so," I would be terribly disappointed and would wonder whether you were a superficial reader or I an ineffectual writer.

The teaching of this book, if teaching there is at all, lies not alone in the information which it presents, but also in the point of view which it invites you to share. If you were to reply, "It taught me to be more alert for the common features which often tie together seemingly unrelated things," or, "It taught me that Nature is basically simple and consistent to the extent that in facing an apparently complicated phenomenon, I may look first for simple answers suggested by things I already know," then I would be certain that this book had really scored.

Has it?

DR. JOHN N. SHIVE



Physicist John Shive has long been fascinated by similarities in various natural phenomena. In this book he describes these likenesses in the behavior of waves of many kinds. His words strikingly reveal the true simplicity of nature's laws.

Dr. Shive received a Ph.D. in physics from Johns Hopkins University in his native Baltimore in 1939. He then joined Bell Telephone Laboratories where he has done research and development work on semiconductors. The phototransistor is one of his major inventions.

He is a leading author on scientific topics, particularly semiconductors.

Dr. Shive is now Director of Education and Training at Bell Laboratories.

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